# DOING PHYSICS WITH PYTHON 

COMPLEX SYSTEMS<br>\section*{CSI: MURDER - TIME OF DEATH?}<br>Ian Cooper<br>matlabvisualphysics@gmail.com<br>\title{ DOWNLOAD DIRECTORIES FOR PYTHON CODE Google drive }

## GitHub

cs_002.py

## CSI: MURDER - TIME OF DEATH?

At the scene of a crime a dead person was found. Can you determine the time of death?

## What details do you need to know?

What meaurements need to be taken?

The temperature of the body was measured to be $32.0^{\circ} \mathrm{C}$ at 5.00 pm and at $28.5^{\circ} \mathrm{C}$ on hour later at $6: 00 \mathrm{pm}$. The room temeprature was 22 ${ }^{\circ} \mathrm{C}$.

Estimate an uncertainty in the time of death assuming that the room temperature measurement could have fluctuated by $1^{\circ} \mathrm{C}$.

How to solve this problem?

The problem can be solved by using Newton's law of cooling where the law can be expressed as a 1 dof difference equation.

## Newton's law of cooling

The nature of the thermal energy transferred from one place at a higher temperature to another place of lower temperature is complicated and in general involves the processes of conduction, convection and radiation. However, if this temperature difference is not too large, the rate of change of temperature can be approximated
using Newton's law of cooling which states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the its surrounding environment temperature. The 1 dof difference equation for Newton's law of cooling is
(1) $\quad T(t+\Delta t)=T(t)-\Delta t R\left(T(t)-T_{\text {env }}\right)$
where $t$ is the time, $\Delta t$ is the time step (the smaller $\Delta t$ the better), $T_{e n v}$ is the surrounding environmental temperature and $R$ is a rate constant.

## The Model

Equation 1 can be used to find the time of death of the murdered person. We know that the environmental temperature is $T_{\text {env }}=(22 \pm 1)$ ${ }^{\circ} \mathrm{C}$. We assume the body temperature at the time of death is $37^{\circ} \mathrm{C}$ and we know that at 5:00 pm the body temperature was $32^{\circ} \mathrm{C}$ and at 6:00 pm it was $28.5^{\circ} \mathrm{C}$. So, in 60 minutes, the body temperature fell from $32{ }^{\circ} \mathrm{C}$ to $28.5^{\circ} \mathrm{C}$.

The first step is to find the value of the rate constant $R$ for each environmental temperature 21,22 and $23^{\circ} \mathrm{C}$ by solving the equation with the initial condition $T_{0}=37^{\circ} \mathrm{C}$ for a time span from $t=0$ to $t$ $=140 \mathrm{~min}$. The value of $R$ is found by a trail-and-error approach, so just start with a sensible guess.

Second step is to plot the graph of the fall in body temperature against time and very the value of $R$ until the time interval for the temperature
to drop from $32{ }^{\circ} \mathrm{C}$ to $28.5^{\circ} \mathrm{C}$ is 60 minutes. Find $R$ for each of the three environmental temperatures.

Third step is to find the time it takes the for the body temperature to drop from $37^{\circ} \mathrm{C}$ to $32^{\circ} \mathrm{C}$ for each the values of $R$ found for each environmental temperature. From this time interval, you then calculate the time of the murder and its uncertainty.

## Simulations

Figure 1 shows the $T$ vs $t$ plot for $R=6.800 \times 10^{-3}$ and $T_{\text {env }}=21^{\circ} \mathrm{C}$.
The time interval for the temperature to drop from $32{ }^{\circ} \mathrm{C}$ to $28.5^{\circ} \mathrm{C}$ is $56.25 \mathrm{~min}<60 \mathrm{~min}$. So, the temperature falls to rapidly, implying the value of $R$ needs to be decreased.


Fig. 1.

Figure 2 shows the result for the correct value of $R$ where the time interval for the temperature to drop from $32{ }^{\circ} \mathrm{C}$ to $28.5^{\circ} \mathrm{C}$ is 60.04 min . From figure 2 the time interval for the temperature to drop from $37^{\circ} \mathrm{C}$ to $32^{\circ} \mathrm{C}$ is 58.8 min . Therefore, the time of the murder is predicted to be at 4:01 pm.


Fig. 2.

Repeating the proceed gives the estimated time of death to be between 3:59 to 4:07 pm.

