DOING PHYSICS WITH PYTHON

[2D] NON-LINEAR DYNAMICAL SYSTEMS

SYSTEMS WITH REAL EIGENVALUES

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Please email me any corrections, comments, suggestions or additions: **matlabvisualphysics@gmail.com**

DOWNLOAD DIRECTORIES FOR PYTHON CODE

[Google drive]

[GitHub]

cs210.py

Reference

Stephen Lynch

Dynamical Systems with Applications using Python

Example **cs210.py**

System equations

$$\dot{x} = x \qquad \dot{y} = x^2 + y^2 - 1$$

Fixed points $x_e = 0$, $y_e = \pm 1$

(0, -1) and (0, +1)

Jacobian matrix

$$f(x,y) = x \quad g(x,y) = x^2 + y^2 - 1$$

$$\partial f / \partial x = 1 \quad \partial f / \partial y = 0 \quad \partial g / \partial x = 2x \quad \partial g / \partial y = 2y$$

$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 2x & 2y \end{pmatrix}$$

Stability

Fixed point (0, -1)

$$\mathbf{J}(0,-1) = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$$

Eigenvalues $\lambda_0 = 1 \quad \lambda_1 = -2$

Eigenvalues real: one positive and one negative

$\Rightarrow$ fixed point is a **saddle point**

Fixed point (0, +1)

$$\mathbf{J}(0,+1) = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

Eigenvalues $\lambda_0 = 1 \quad \lambda_1 = 2$

Eigenvalues real: both positive

$\Rightarrow$ fixed point is **unstable**

A summary of the system parameters and results are displayed in the Console.

Nullclines

$$\dot{x} = x \qquad \dot{y} = x^2 + y^2 - 1$$

x-nullcline: $\dot{x} = 0 \Rightarrow x = 0$

y-nullcline: $\dot{y} = 0 \Rightarrow x^2 + y^2 = 1$

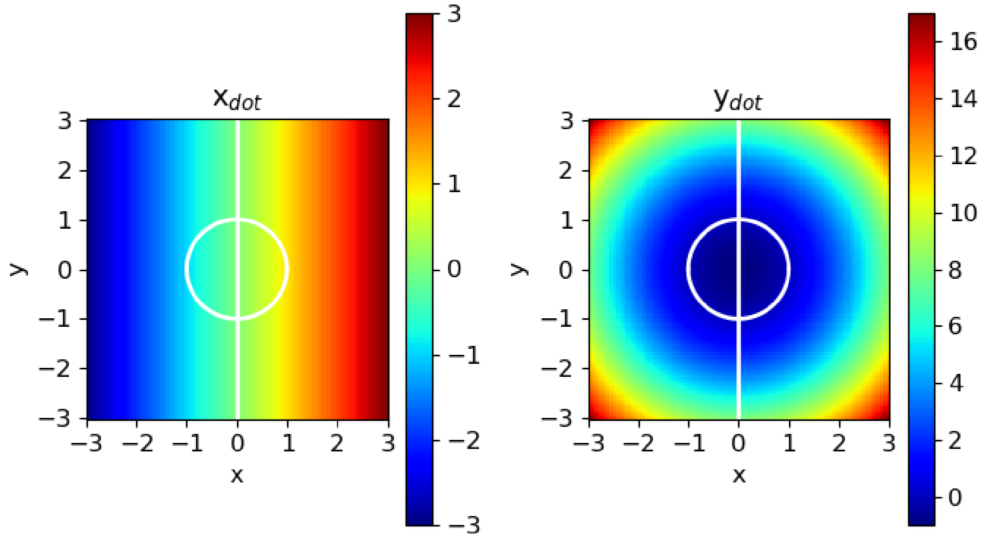

Fig 1.1. [2D] view of the system equations: nullclines (white). Intersection of nullclines gives the fixed points (0,-1) and(0,+1).

$$\dot{x} = x \qquad \dot{y} = x^2 + y^2 - 1$$

x_{dot}

$$x > 0 \Rightarrow \dot{x} > 0 \ \rightarrow$$

$$x < 0 \Rightarrow \dot{x} < 0 \ \leftarrow$$

y_{dot}

Inside the circle: $x^2 + y^2 - 1 < 0$ flow in y direction is towards $y = 0$ and the flow in the x direction is always away from the y-axis ($x = 0$).

Outside circle: $x^2 + y^2 - 1 > 0$ flow in y direction is away from $y = 0$ and the flow in the x direction is always away from the y-axis ($x = 0$).

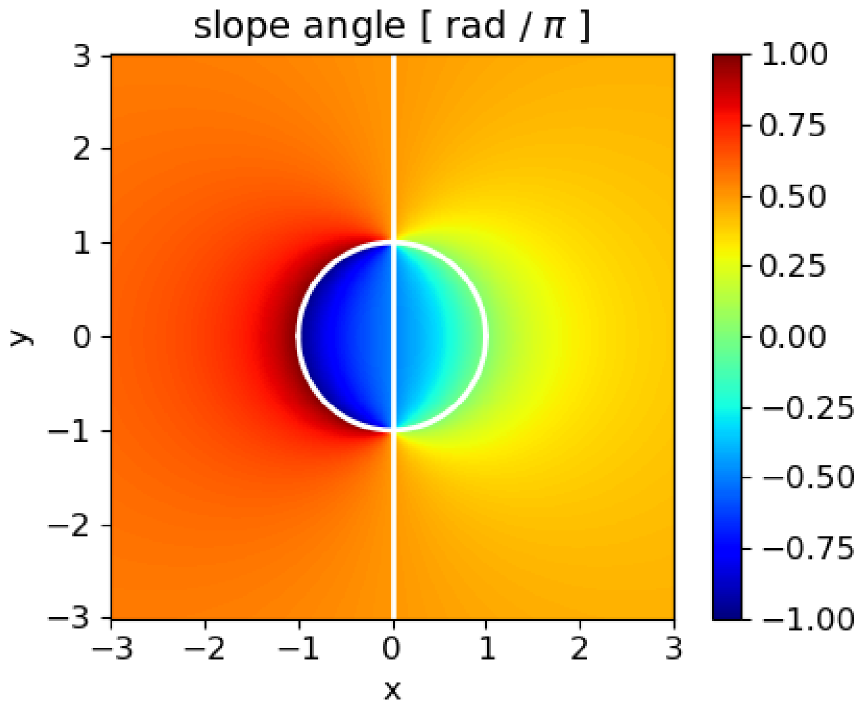

Fig. 1.2. Slope angle θ.

$\theta = 0 \rightarrow$ $\theta = 0.5 \uparrow$ $\theta = -0.5 \downarrow$ $\theta = -1 \leftarrow$ $\theta = +1 \leftarrow$

The slope function and its slope angle are $dy(x, y)/dx = \tan\theta$ where θ is expressed in rad/π. Therefore $-1 \le \theta \le +1$.

Below are a set of plots with different initial conditions.

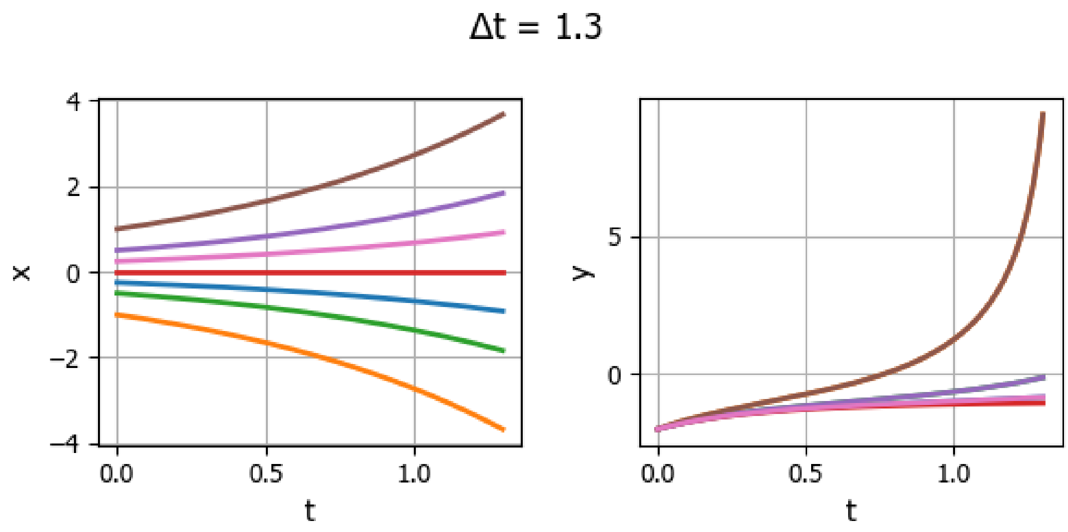

Fig. 1.3. Trajectories for different initial conditions in the time interval $\Delta t = 1.30$. For the initial condition (1,-2), the trajectory rapidly diverges to infinity when $\Delta t > 1.30$,

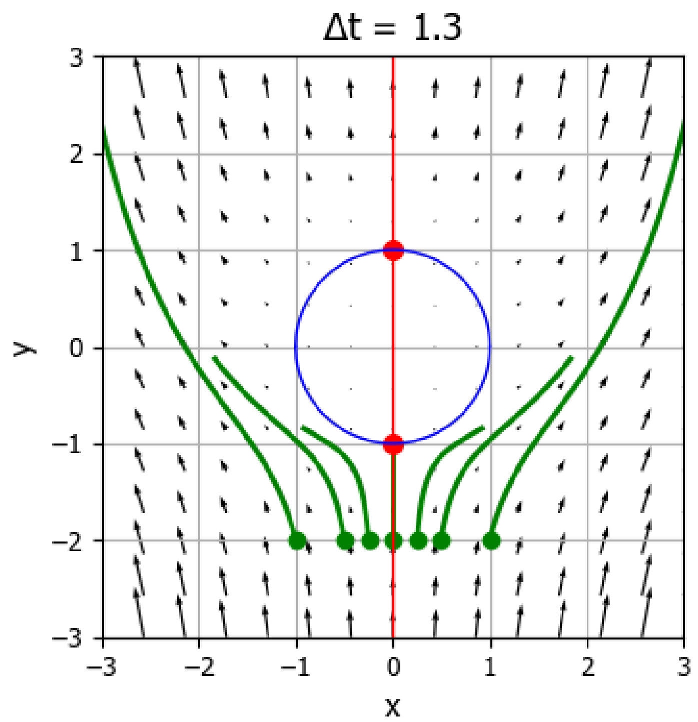

Fig. 1.4. Phase portrait (quiver plot). The red dots show the critical points **unstable** (0,1), and **saddle** (0,-1). The red vertical line is the x-nullcline and the blue circle is the y-nullcline.

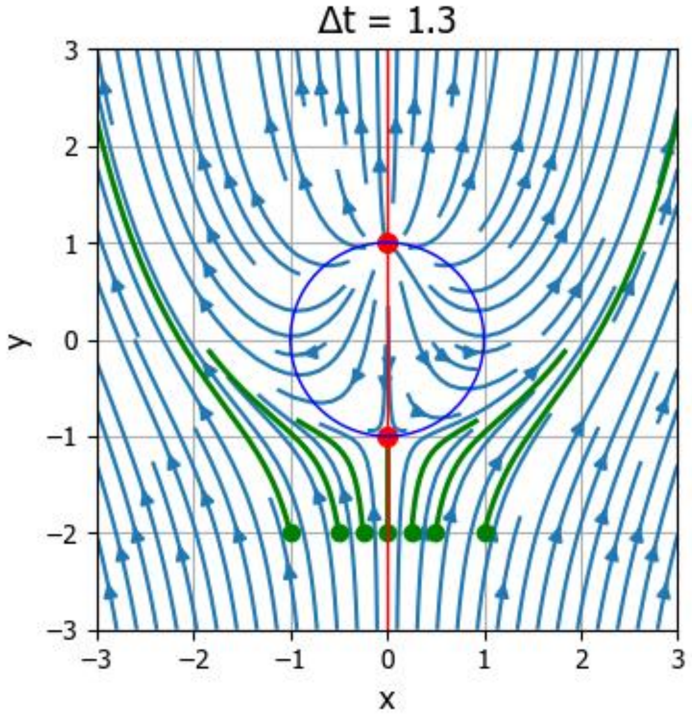

Fig. 1.5. Phase portrait (streamplot). The red dots show the critical points **unstable** (0,1) and **saddle** (0,-1). The red vertical line is the x-nullcline and the blue circle is the y-nullcline. The streamplot makes it very easy to predict the trajectory from any staring point.

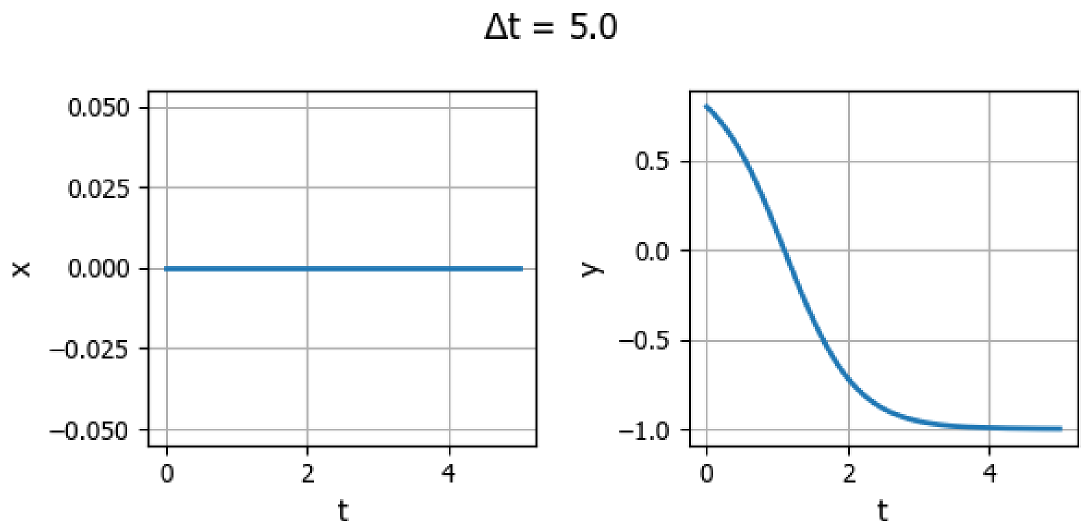

Fig. 1.6.

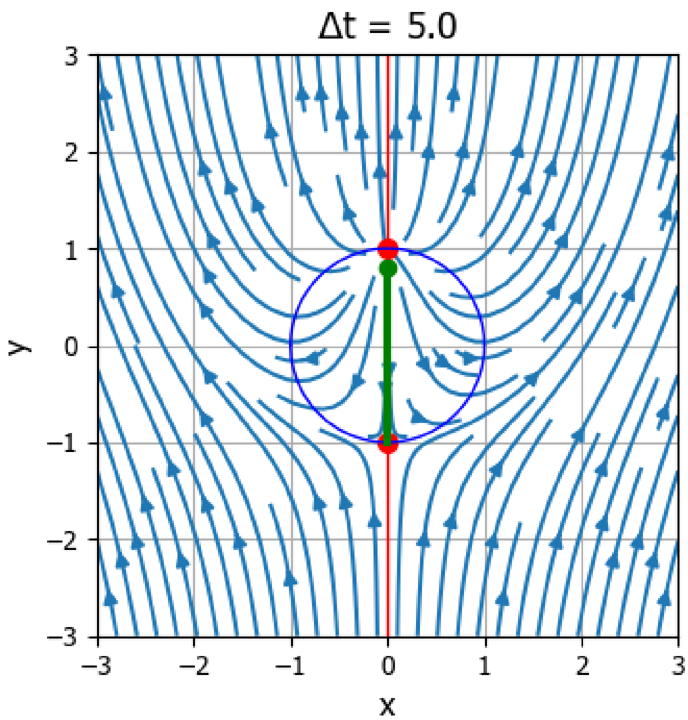

Fig. 1.6. Trajectory for the initial condition (0,0.8).

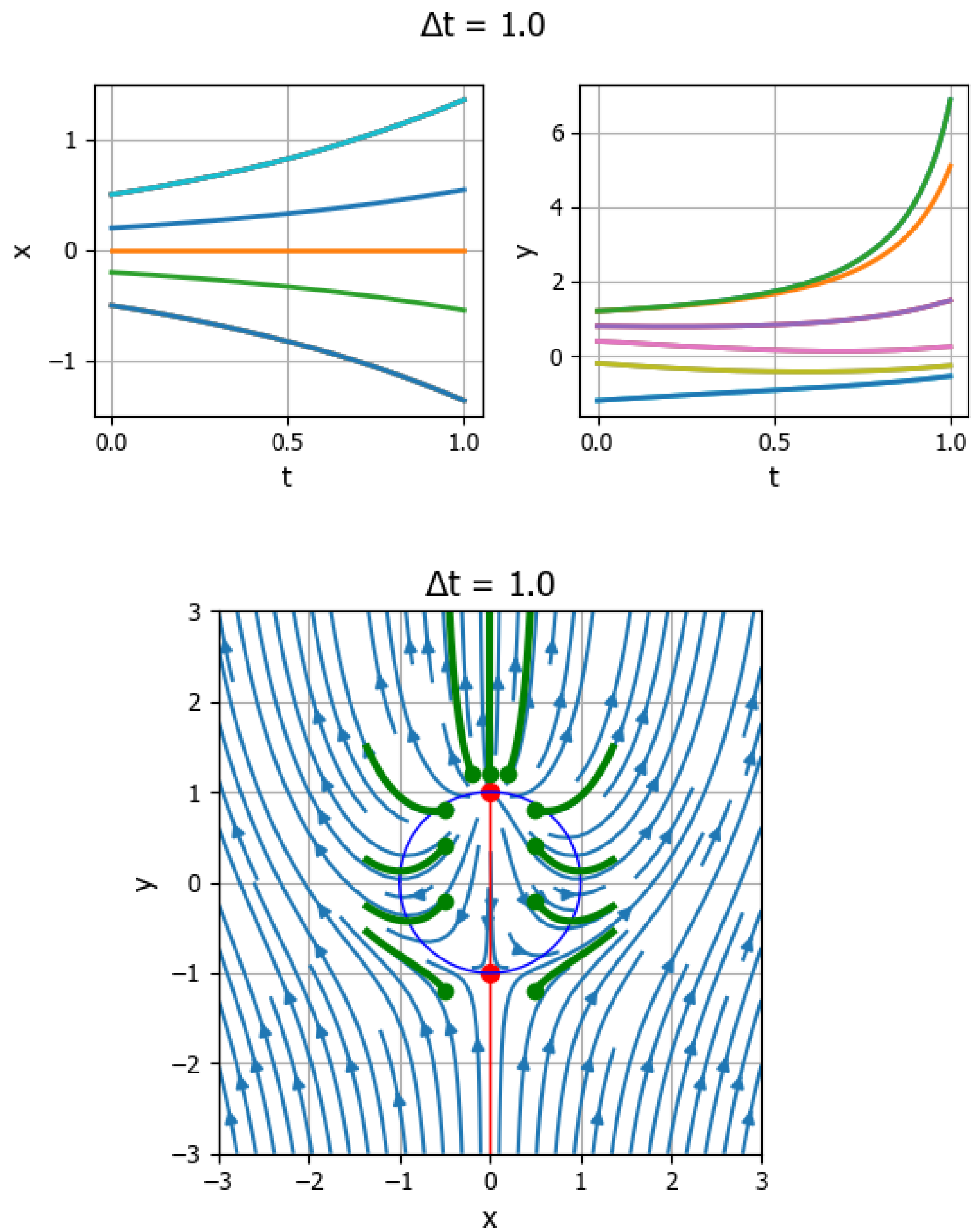

Fig. 1.7. The critical point (0,1) is **unstable** whereas the critical point (0,-1) is a **saddle**.