DOING PHYSICS WITH PYTHON

[2D] NON-LINEAR DYNAMICAL SYSTEMS

SYSTEM WITH COMPLEX EIGENVALUES

Ian Cooper

Please email me any corrections, comments, suggestions or additions: **matlabvisualphysics@gmail.com**

DOWNLOAD DIRECTORIES FOR PYTHON CODE

Google drive

GitHub

cs211.py

Reference

Stephen Lynch

Dynamical Systems with Applications using Python

Example **cs211.py**

System equations

$$\dot{x} = y \qquad \dot{y} = x\left(1 - x^2\right) + y$$

Jacobian matrix $\mathbf{J} = \begin{pmatrix} 0 & 1 \\ 1 - 3x^2 & 1 \end{pmatrix}$

x-nullcline $\dot{x} = 0 \Rightarrow y = 0$

y-nullcline $\dot{y} = 0 \Rightarrow y = x\left(x^2 - 1\right)$

There are three fixed (critical) points.

Critical point (0, 0)

$$\mathbf{J}(0,0) = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

Eigenvalues **[-0.618 1.618]**

Eigenvectors $\begin{pmatrix} -0.851 & -0.526 \\ 0.526 & -0.851 \end{pmatrix}$

The critical point at the Origin **(0, 0)** is a **saddle point** as both eigenvalues are real, one positive and one negative.

Critical point (1, 0)

$$\mathbf{J}(1,0) = \begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$$

Eigenvalues **[0.5+1.323j 0.5-1.323j]**

Eigenvectors $\begin{pmatrix} 0.204 - 0.54j & 0.204 + 0.54j \\ 0.816 + 0.j & 0.816 - 0.j \end{pmatrix}$

The critical point at **(1, 0)** is an **unstable focus (spiral point)** since the eigenvalues are complex with the real parts greater zero.

Critical point (-1, 0)

$$\mathbf{J}(-1,0) = \begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$$

Eigenvalues **[0.5+1.323j 0.5-1.323j]**

Eigenvectors $\begin{pmatrix} 0.204-0.54j & 0.204+0.54j \\ 0.816+0.j & 0.816-0.j \end{pmatrix}$

The critical point at **(-1, 0)** is an **unstable focus (spiral point)** since the eigenvalues are complex with the real parts greater zero.

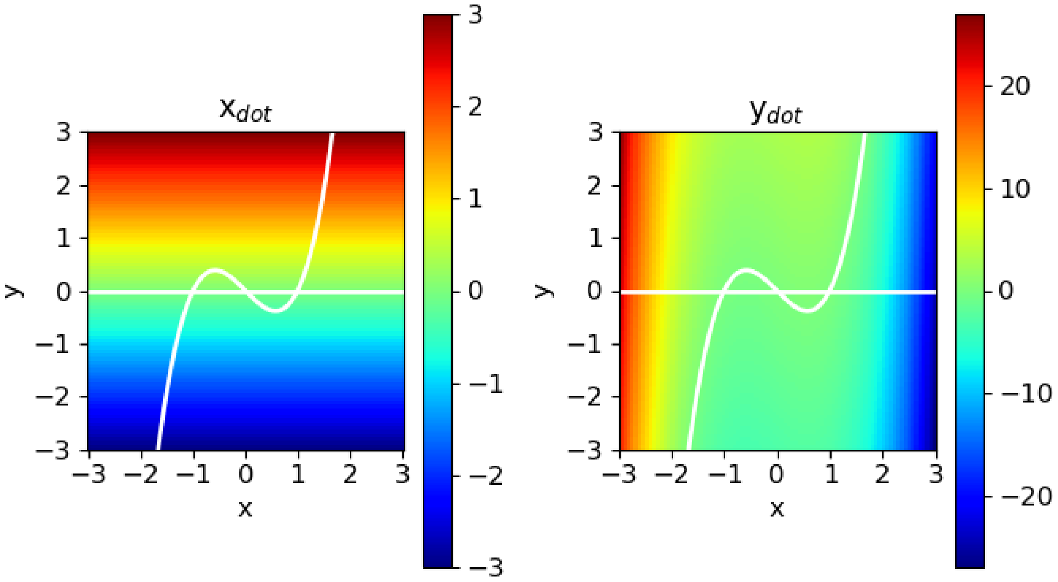

Fig 1. [2D] view of the system equations (nullclines: white lines). x_{dot}: the flow is away from the x axis. y_{dot}: left side flow is + y direction and the flow is in the -y direction on the right.

$\dot{x} = y \qquad \dot{y} = x\left(1 - x^2\right) + y$

x-nullcline $\qquad \dot{x} = 0 \Rightarrow y = 0$

y-nullcline $\qquad \dot{y} = 0 \Rightarrow y = x\left(x^2 - 1\right)$

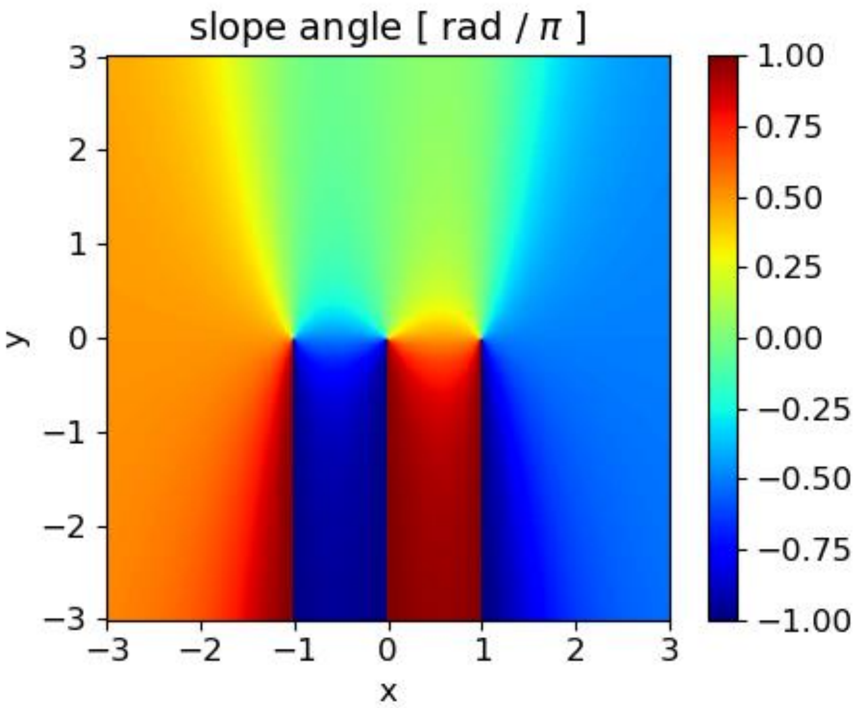

Fig. 2. Slope angle θ.

$\theta = 0 \rightarrow \qquad \theta = 0.5 \uparrow \qquad \theta = -0.5 \downarrow \qquad \theta = -1 \leftarrow \qquad \theta = +1 \leftarrow$

The slope function and its slope angle are $dy(x, y) / dx = \tan\theta$ where θ is expressed in rad/π. Therefore $-1 \le \theta \le +1$.

Below are a set of plots with different initial conditions.

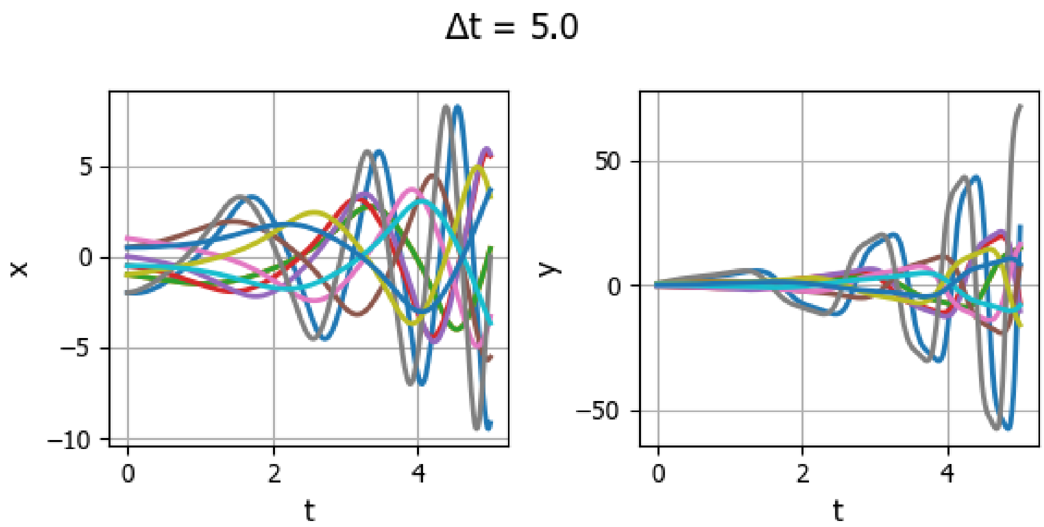

Fig. 3. Trajectories for different initial conditions in the time interval $\Delta t = 5.0$.

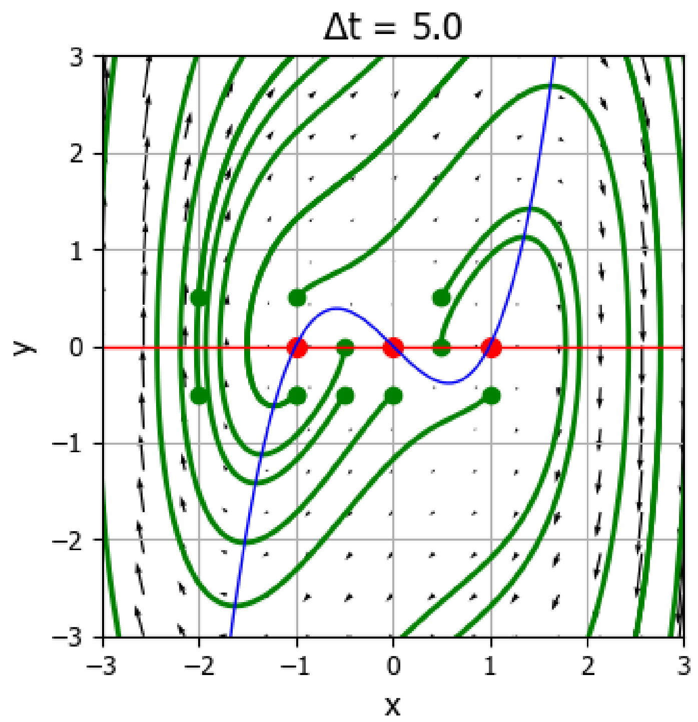

Fig. 4. Phase portrait (quiver plot). The red dots show the critical points (0, 0), (1,0) and (-1,0). The red line is the x-nullcline and the blue line is the y-nullcline.

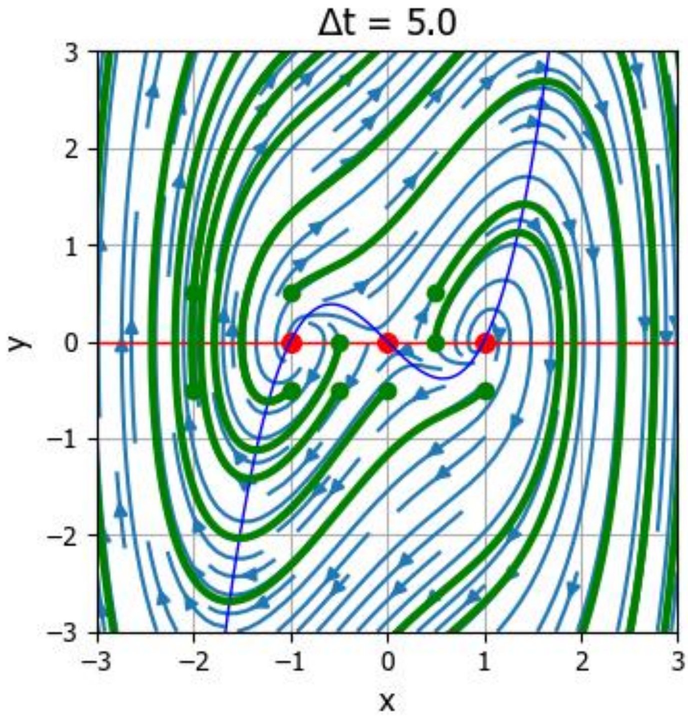

Fig. 5. Phase portrait (streamplot). The red dots show the critical points (0, 0), (1,0) and (-1,0). The red line is the x-nullcline and the blue line is the y-nullcline. The streamplot makes it very easy to predict the trajectory from any staring point. One can observe the **spiral patterns** around the critical points (-1,0) and (1,0).