DOING PHYSICS WITH PYTHON

DYNAMICAL SYSTEMS [1D]

INTRODUCTION

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DOWNLOAD DIRECTORIES FOR PYTHON CODE

Google drive

GitHub

ds25L1.py Damped driven mass-spring system

Jason Bramburger

History and Preliminaries - Dynamical Systems | Lecture 1

https://www.youtube.com/watch?v=Gw19VCtHYcs

INTRODUCTION

As an introduction to dynamical system, an oscillating mass-spring system will be considered. A dynamical system can be modelled by a set of differential equations and the behaviour of the system is completely described by the solutions of the differential equations. The mass-spring system is an example of an initial value problem since you need to know the equations and the initial conditions. The equations are expressed in a set of state variables $x_1, x_2, x_3, \ldots$

Mass – Spring system

Simple Harmonic Motion (SHM)

Damped Harmonic Motion (DHM)

Forced Harmonic Motion (FHM)

The governing equation for the oscillating mass-spring system (no external forcing) is

(1) $$m\ddot{x} + b\dot{x} + kx = 0$$

where m is the mass of the oscillating object, b is the damping constant and k is the spring constant. The state variable x is the displacement from the equilibrium position ($x = 0$) along the X-axis. The equilibrium (steady-state) position is the only fixed point where $x_{ss} = 0$. This is autonomous equation since it does have any terms which are an explicit function of time t.

Equation 1 is a 2nd order ODE and to solve it, the equation is expressed as a set of two 1st order ODEs where x_1 and x_2 are the state variables

(2) $$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -(b/m)x_2 - (k/m)x_1 \end{aligned}$$

and the initial conditions are $x_1(0)$ and $x_2(0)$. The state variable x_1 is the displacement x of the oscillating object from the Origin ($x = 0$), x_2 is velocity v of the object, and $\dot{x}_2$ is the objects acceleration a.

The natural frequencies and period for simple harmonic motion (SHM) $b = 0$ are

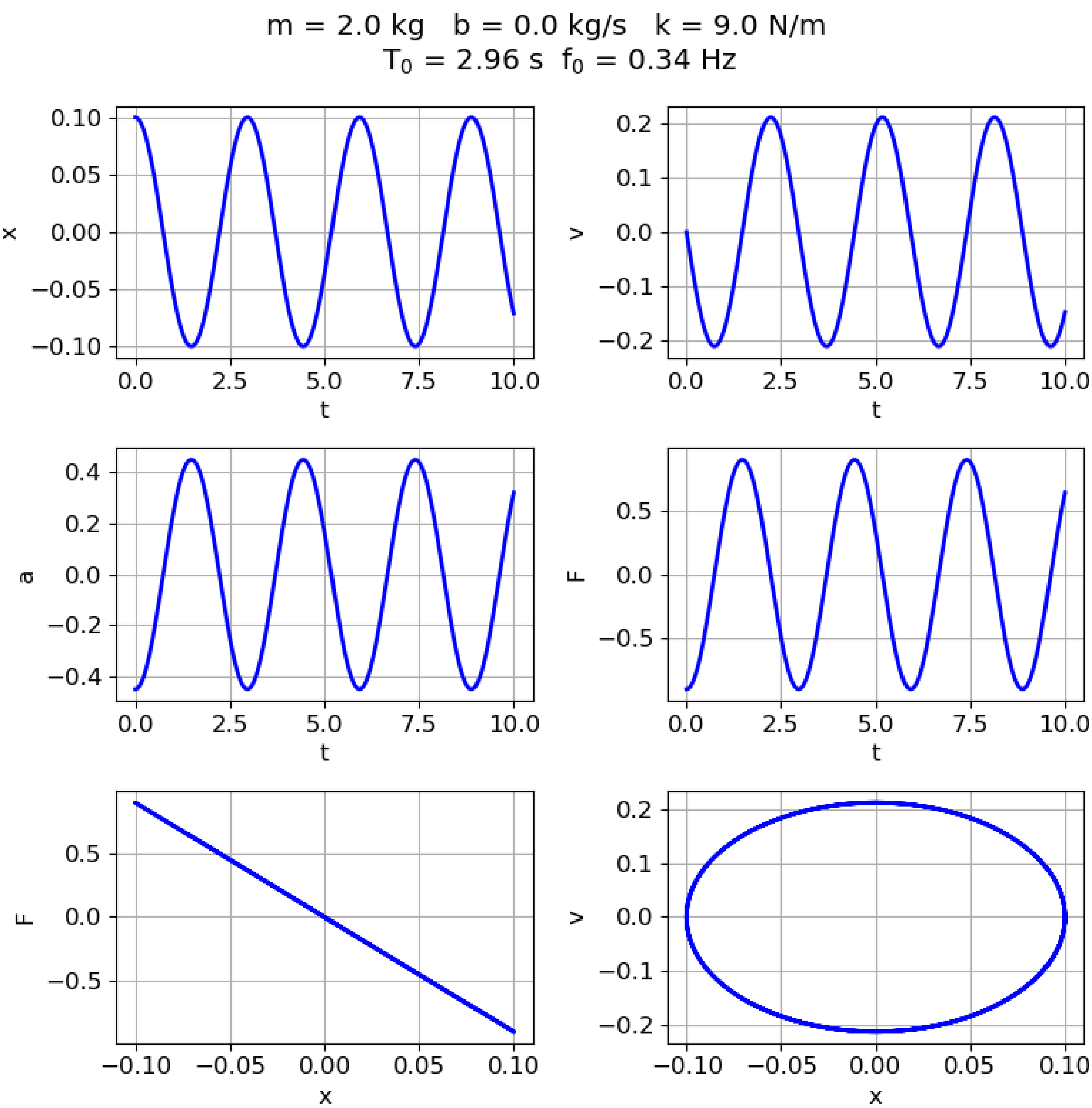
$$(3) \quad \omega_0 = \sqrt{\frac{k}{m}} \qquad f_0 = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

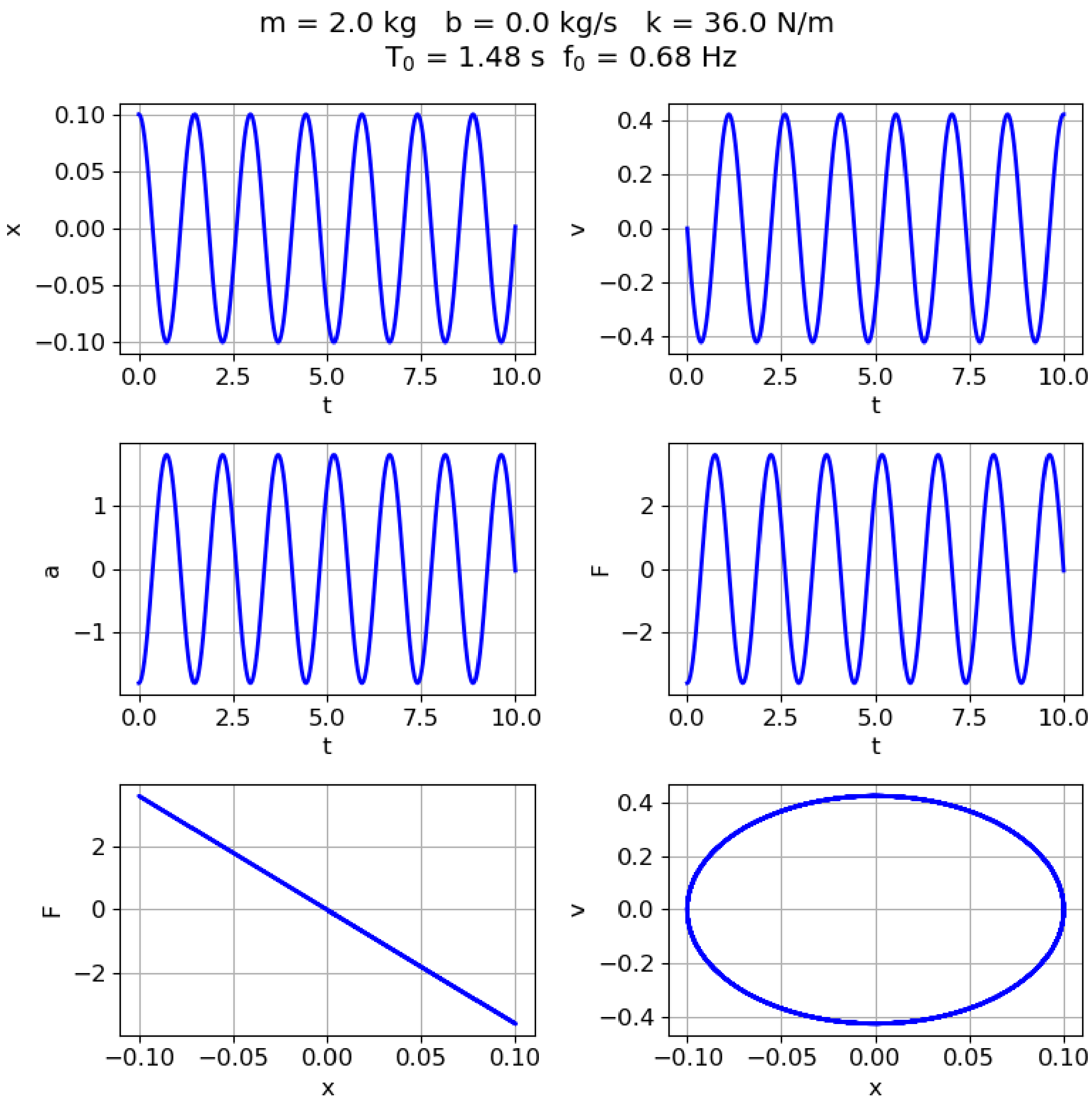
The ODEs can be solved very easily using the Python function **odeint**. The output of the Code is displayed graphically.

 # ds25L1.py

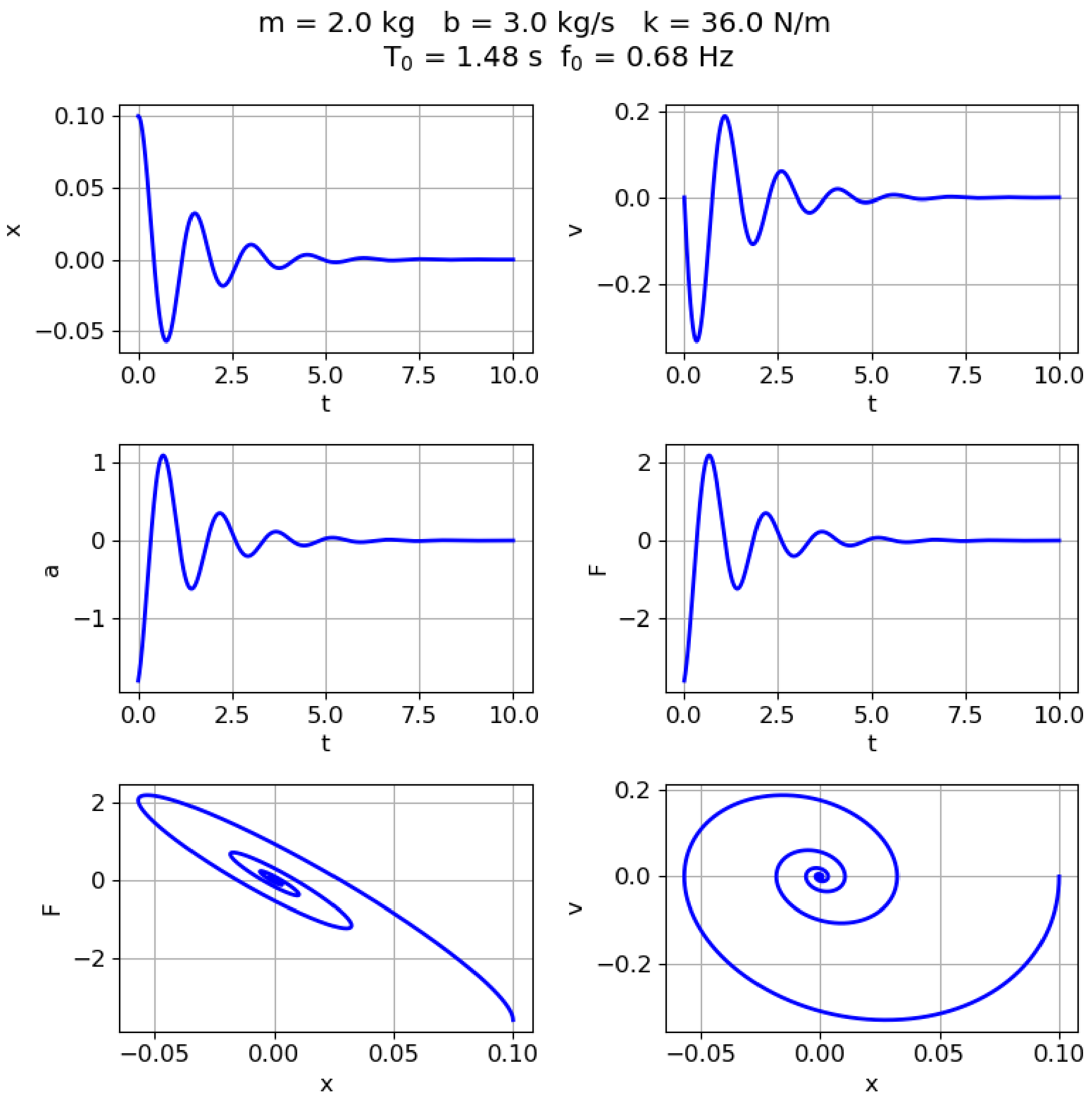
```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
from scipy.signal import find_peaks
import time
from numpy import pi, sin, cos, linspace, sqrt

plt.close('all')

tStart = time.time()

#%%  SOLVE ODE x
def lorenz(t, state):
    x1, x2 = state
    dx1 = x2
    dx2 = -(b/m)*x2 - (k/m)*x1
    return dx1, dx2

#%% SETUP
u0 = [0.1,0]
tMax = 10; N = 9999
m = 2
b = 0.5
k = 9
```

```
w = sqrt(k/m)
T = 2*pi/w
f = 1/T

#%% SOLVE ODE
t = linspace(0,tMax,N)
sol = odeint(lorenz, u0, t, tfirst=True)
x = sol[:,0]
v = sol[:,1]
a = -(b/m)*v - (k/m)*x
F = m*a
```





Increasing the value of *k* from 9 to 36 results in an increase in the frequency and a decrease in the period of oscillation.



Damping: The oscillations decay until the object comes to rest at the Origin $x = 0$.

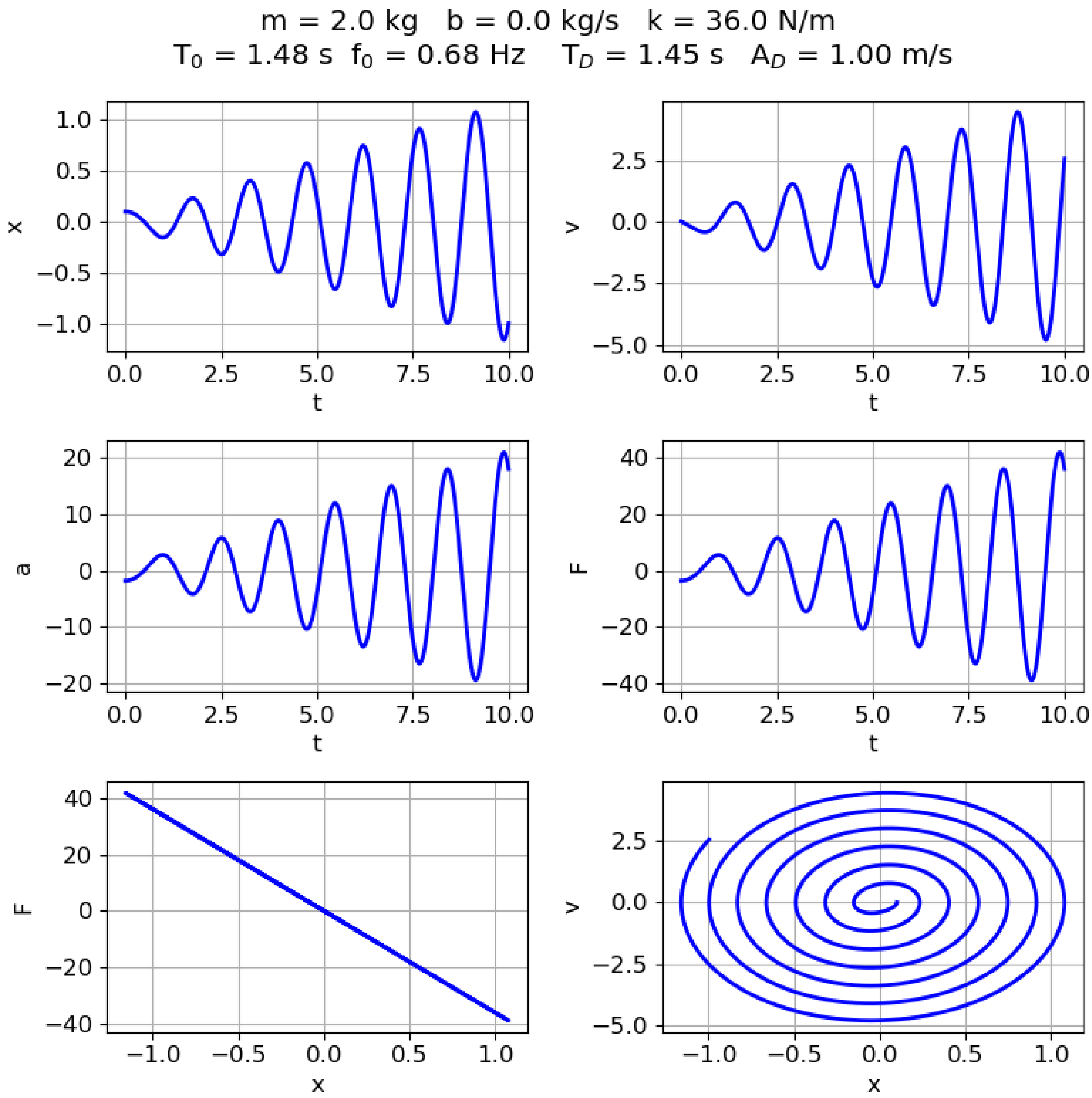
By examining the x vs F graph, we see that the Origin acts as a basin of attraction because the force acting on the object is always pulling it towards the Origin.

When an external sinusoidal force acts on the mass-spring system, then the equation of motion must include a time dependent term and the equation is an now a nonautonomous equation

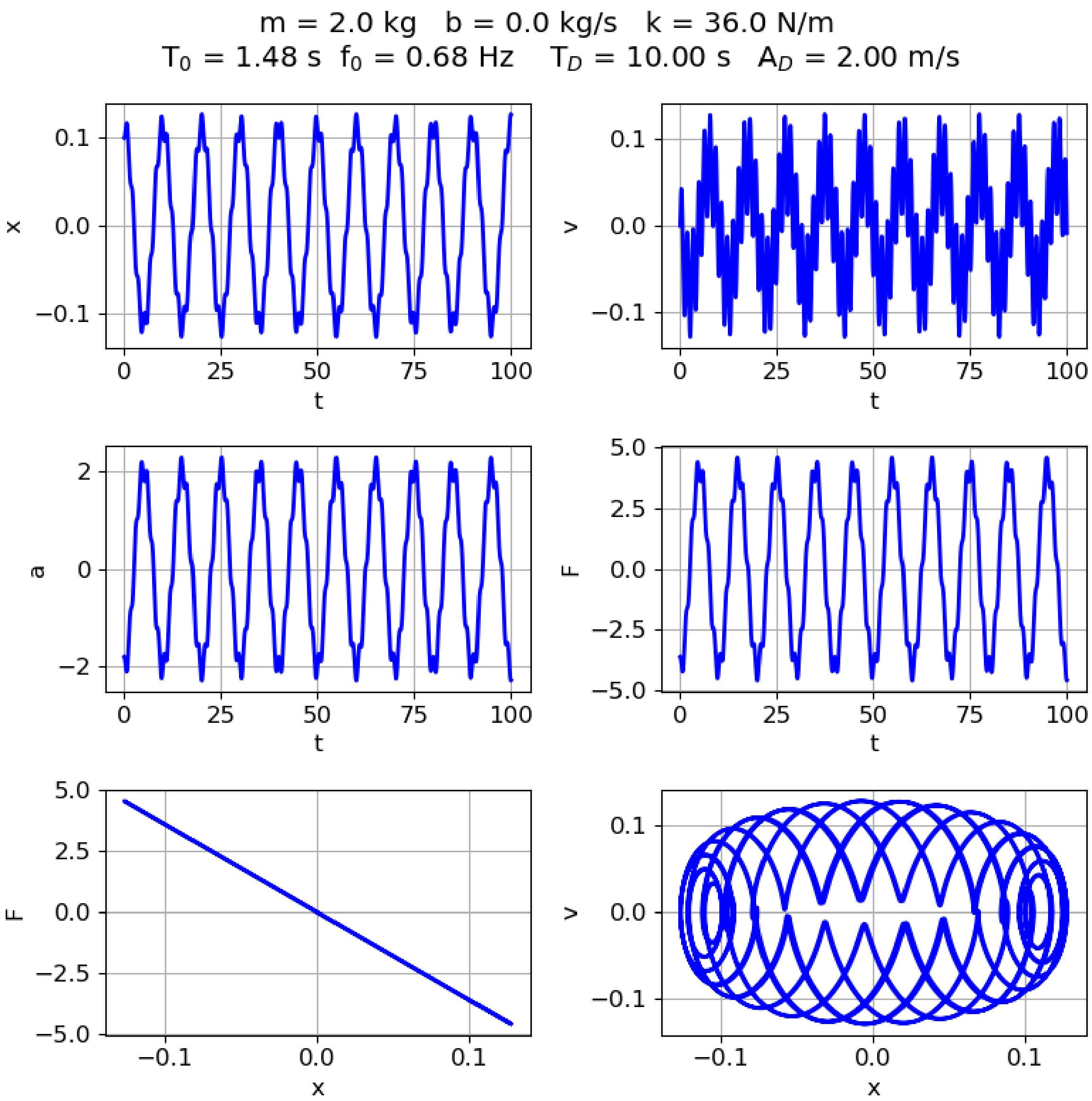
$$(3) \qquad m\,\ddot{x} + b\,\dot{x} + k\,x = A_D \cos(\omega_D\, t)$$

where A_D is the strength of the driving force and the driving frequency is f_D $\left(\omega_D = 2\pi\, f_D\right)$. The set of 1st order ODEs are

$$(4) \qquad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\left(b/m\right)x_2 - \left(k/m\right)x_1 + A_D\cos(\omega t) \end{aligned}$$



$b = 0 \quad f_D \approx f_0 \;\Rightarrow$ oscillations grow and grow (resonance)



When the driving frequency f_D is not near the natural frequency f_0, then oscillations do not grow and the system oscillates near the driving frequency.

View

[Chaotic dynamical systems: a damped driven pendulum]