DOING PHYSICS WITH PYTHON

DYNAMICAL SYSTEMS [1D]

MODELLING FIREFLY ENTRAINMENT

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DOWNLOAD DIRECTORIES FOR PYTHON CODE

Google drive

GitHub

ds25L13.py

Jason Bramburger
Modelling Firefly Entrainment - Dynamical Systems | Lecture 13

https://www.youtube.com/watch?v=4Bp81-yU6k4

INTRODUCTION

In this paper we explore and application of dynamical systems on the circle. We model the entrainment of a firefly due to an external periodic stimulus. We demonstrate that there is a range of parameter values for which the firefly synchronizes its flashing with the stimulus, but if the stimulus is too fast or to slow the firefly will not be to catch up. A result is a range of entrainment for which we can flash the stimulus to get the firefly to sync up with us.

Modelling firefly entrainment uses dynamical systems to understand how individual fireflies synchronize their light flashes with external stimuli or other fireflies. Models describe fireflies as pulse-coupled oscillators, where the frequency of an external stimulus causes the oscillator (the firefly's flashing mechanism) to match it, provided the stimulus frequency is within a certain range. This process, called entrainment, allows for collective behaviours like synchronized flashing where a firefly's natural oscillation is synchronized to an external periodic stimulus, such as another firefly's flash or a man-made light. For entrainment to occur, the external stimulus's frequency (forcing frequency) must be close to the firefly's natural flashing frequency. This is the outcome of entrainment, where multiple fireflies synchronize their flashes, leading to coordinated displays of light.

How our models works

The models predict that if the stimulus is too fast or too slow, a firefly will not synchronize and will continuously try to "catch up" to the stimulus.

A firefly's flashing is modelled as a rotating vector with angular displacement x_F and angular velocity $\dot{x}_F$ and the oscillating external stimulus as a rotating vector with angular displacement x_S and angular velocity $\dot{x}_S$. (Roman characters are used and not Greek to match the variable names in the Python Code).

The model that is used is based upon the flow on a circle where

$$\dot{x}_F = w_F + A\sin\left(x_S - x_F\right)$$

where ω_F is the natural frequency of the firefly flashing (w_F ~0.9 Hz), and $A > 0$ is a measure of the response time of the firefly to the external stimulus. x_F is a firefly flashing rhythm and every time $x_F(t) = 0$ there is a flash and when $x_S(t) = 0$ there is a flash from the external stimulus at a frequency w_S.

$$w_F > 0 \quad w_S > 0$$

To investigate the entrainment, we need to consider the phase difference x between the two rotating vectors

$$x = x_S - x_F$$

So,

$$\dot{x} = \dot{x}_S - \dot{x}_F = w_S - w_F - A\sin(x)$$

Only when the phase difference x between the firefly flashing and the external stimulus flashing do we get synchronization.

$x = x_S - x_F = 0$ entrainment

$x = x_S - x_F \neq 0$ fire fly flashes either lags or leads external stimulus or no stable phase difference

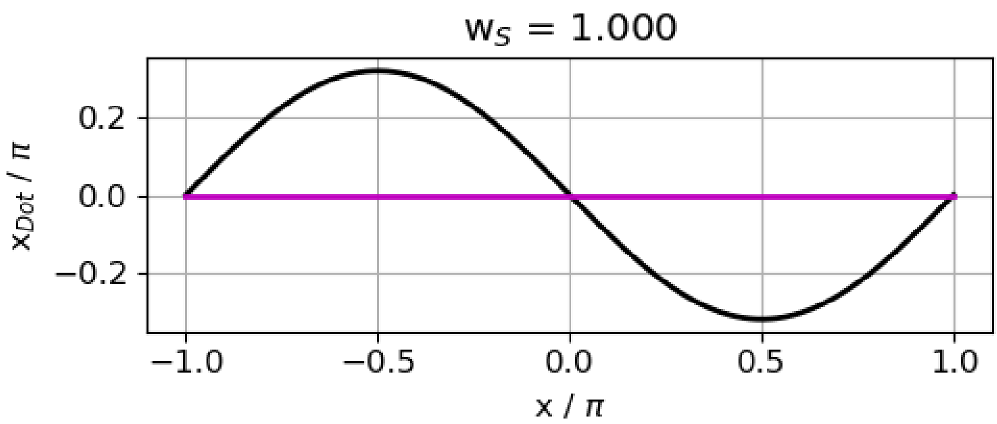
SIMULATIONS

Default values $A = 1.0$ $w_F = 1.0$

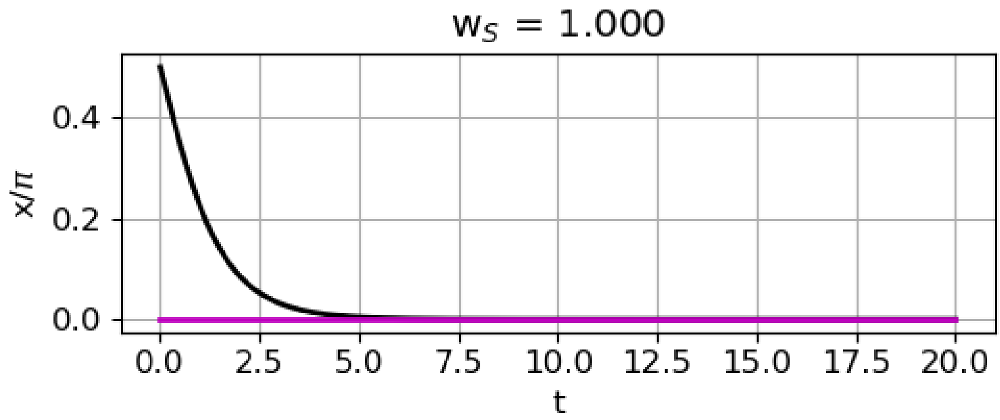
w_S is the adjustable parameter

$\boldsymbol{w_S = w_F}$

When the frequency of the external stimulus w_S matches the natural flash frequency w_F of the firefly, the phase difference x is locked into zero $(x_{SS} = 0)$ and the flashes are synchronized.

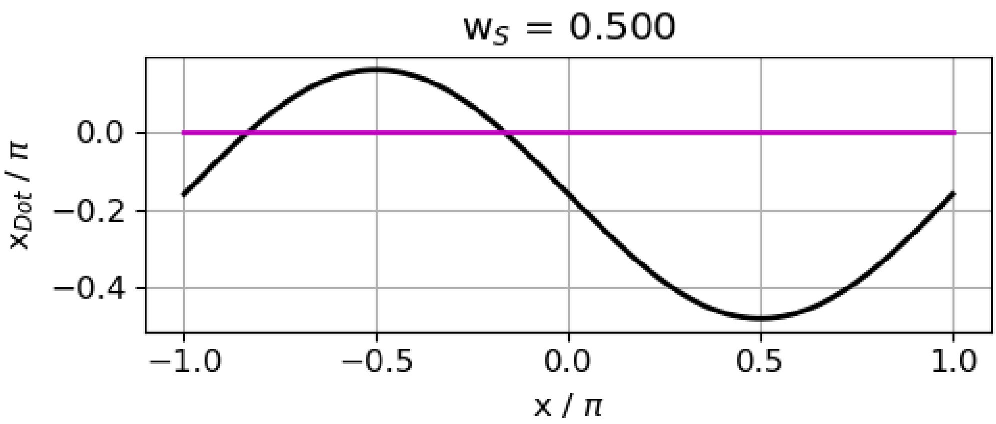


The stable fixed point occurs when the phase difference $x = x_{SS} = 0$.

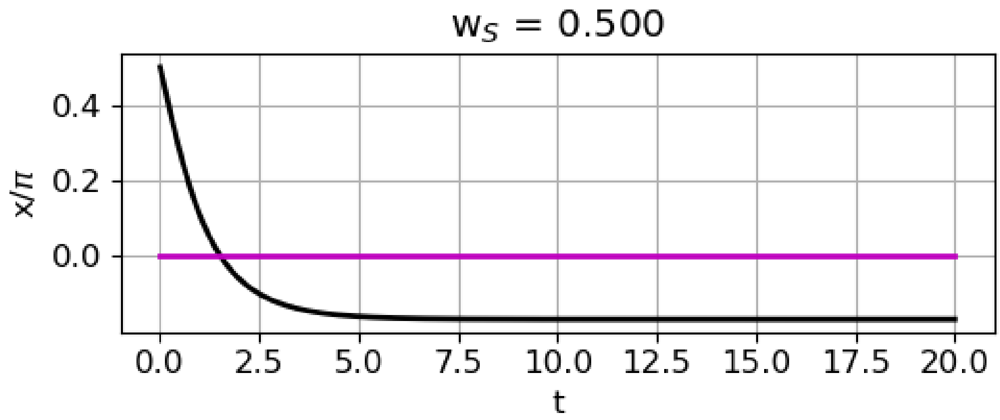


The phase difference x locks into the stable fixed point $x_{SS} = 0$

$w_S < w_F$

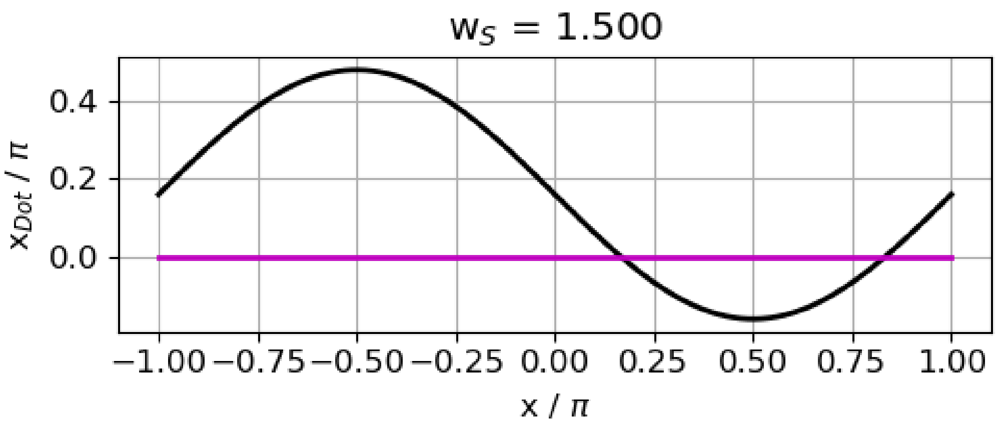


The stable fixed point is $x_{SS} = -0.167\,\pi$ and the phase different is locked into $x = x_{SS} = -0.167\,\pi$ so that the firefly flashes lead the external stimulus flashes.

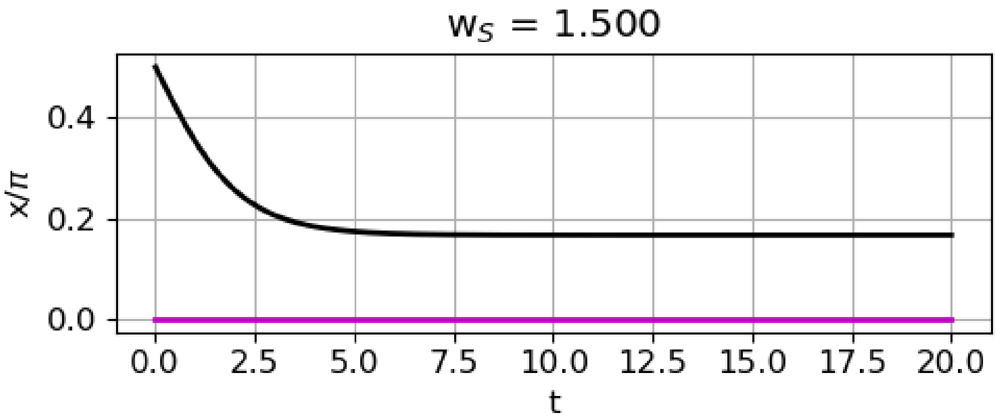


The phase difference x locks into the stable fixed point x_{SS} = - 0.167.

$w_F < w_S < 2\ w_F$

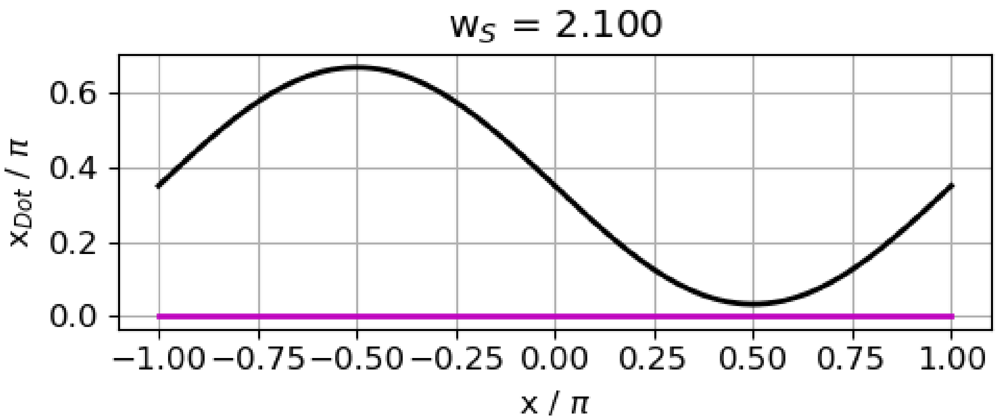


The stable fixed point is $x_{SS} = +0.167\,\pi$ and the phase different is locked into $x = x_{SS} = +0.167\,\pi$ so that the firefly flashes lags the external stimulus flashes.

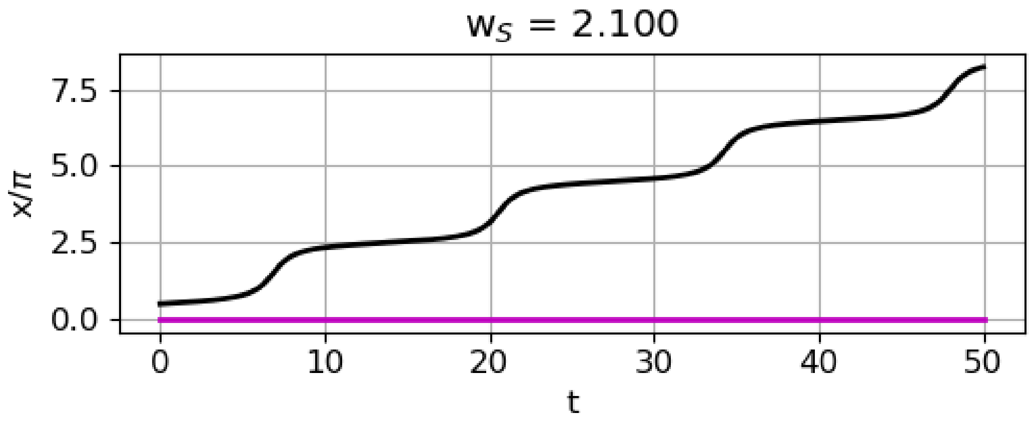


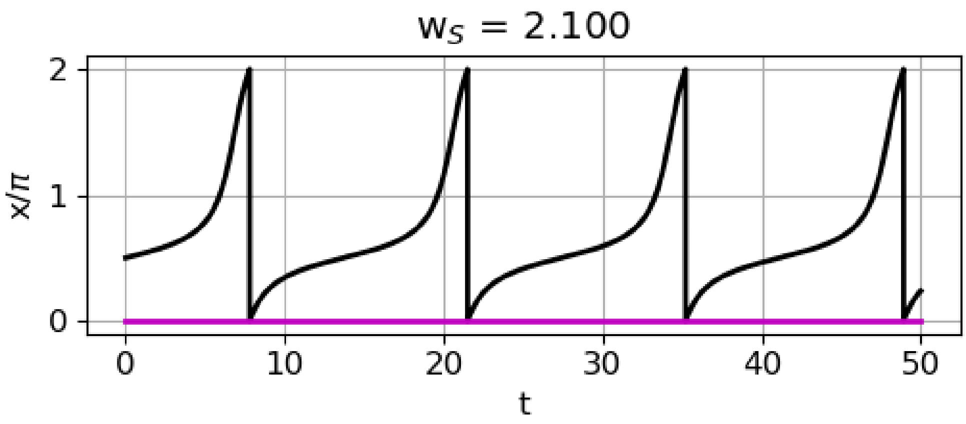
The phase difference x locks into the stable fixed point $x_{SS} = +\ 0.167$.

$w_S > 2\ w_F$



There are now no fixed points. The phase difference x does not lock into any fixed value, so that the phase difference is never constant and there is no synchronization between the flashes.





The phase difference x continually changes. Therefore, the flashes are not synchronized.