DOING PHYSICS WITH PYTHON

DYNAMICAL SYSTEMS [1D]

THE GEOMETRY OF FLOWS ON THE LINE

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DOWNLOAD DIRECTORIES FOR PYTHON CODE

Google drive

GitHub

ds25L2.py $\dot{x} = f(x) = \sin(x)$

Jason Bramburger
The Geometry of Flows on the Line - Dynamical Systems | Lecture 2

https://www.youtube.com/watch?v=LSpXhwsksVY

INTRODUCTION

This article will exam the geometry of flows on the line for [1D] dynamical systems of the form

$$\dot{x} = f(x)$$

In the analysis of such dynamical systems, we can solve the ODE equation to find the time variation of the state variable x, find the steady-state solutions x_{ss} and check their stability.

SIMULATION $\dot{x} = \sin(x)$

(1) $\dot{x} = \sin(x)$ initial condition $x(0) = x_0$

Equation (1) and be solved numerically using the Python function **odeint**.

```
from scipy.integrate import odeint
def lorenz(t, state):
   x = state
   dx = sin(x)
   return dx
#%% SETUP
u0 = pi/2
tMax = 10; N = 9999
#%% SOLVE ODE
t = linspace(0,tMax,N)
sol = odeint(lorenz, u0, t, tfirst=True)
x = sol[:,0]
# fixed points
xss = zeros(2)
xss[0] = 0; xss[1] = pi
# xDot
X = linspace(0,2*pi,999)
Xdot = sin(X)
```

The steady-state solutions are

$$(2) \qquad \dot{x} = \sin(x_{ss}) = 0 \Rightarrow x_{ss} = 0 \text{ and } x_{ss} = \pi$$

where x_{ss} is a fixed-point of the system.

To determine the stability of each fixed point, let

$$f(x) = \sin(x) \qquad f'(x) = \cos(x)$$

then

$f'(x_{ss}) < 0$ **stable fixed point**

$\Rightarrow$ the flow is decreasing and moving to left (-x direction)

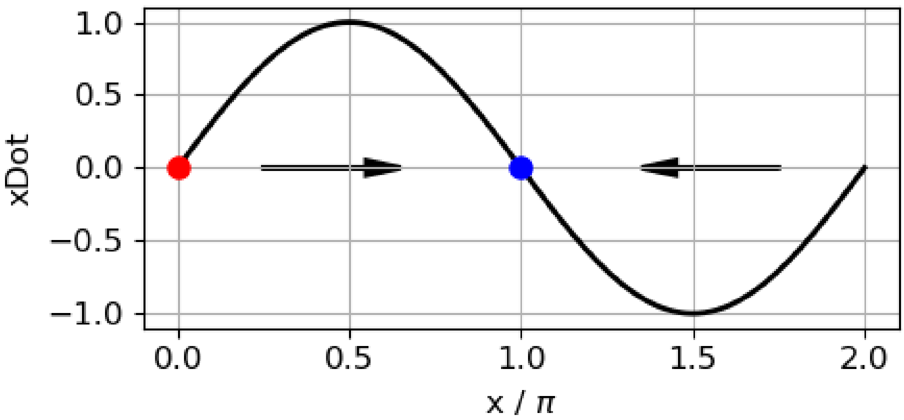
$f'(x_{ss}) > 0$ **unstable fixed point**

$\Rightarrow$ the flow is increasing and moving to right (+x direction)

Thus,

$x_{ss} = \pi \Rightarrow f'(\pi) = \cos(\pi) = -1 < 0$ **stable fixed point**

$x_{ss} = 0 \Rightarrow f'(0) = \cos(0) = 1 > 0$ **unstable fixed point**



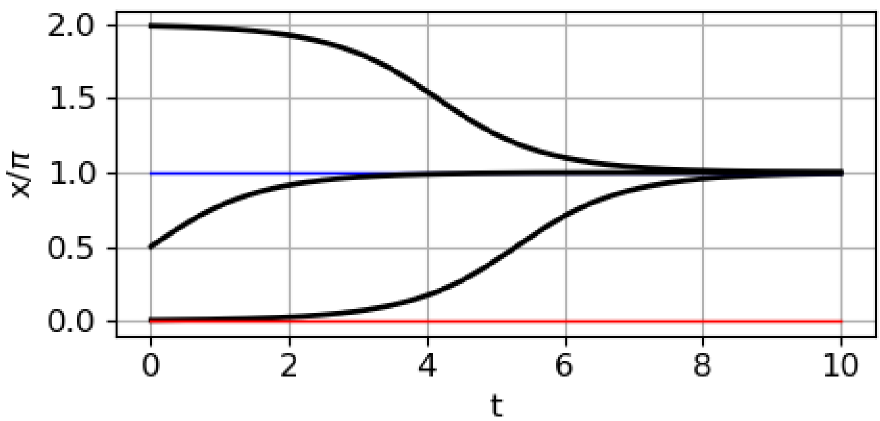
The fixed points are $x_{ss} = 0$ (positive slope, unstable) and $x_{ss} = \pi$ (negative slope, stable).

$0 < x < \pi \;\Rightarrow\; f'(x) > 0 \Rightarrow\; x$ increasing, flow to the right

$\pi < x < 2\pi \;\Rightarrow\; f'(x) < 0 \Rightarrow\; x$ decreasing, flow to the left

$x_{ss} = 0 \quad x_{ss} = \pi \Rightarrow$ no flow

$x_{ss} \neq 0 \quad x_{ss} \neq \pi \Rightarrow t \to \infty \;\; x \to \pi$



$x_{ss} \neq 0 \quad x_{ss} \neq \pi \Rightarrow$ all trajectories are pulled towards the stable fixed point $x_{ss} = \pi$.