DOING PHYSICS WITH PYTHON

DYNAMICAL SYSTEMS [1D]

POTENTIALS

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DOWNLOAD DIRECTORIES FOR PYTHON CODE

Google drive

GitHub

ds25L4.py $\dot{x} = x^2 - 1$

Jason Bramburger
Potentials - Dynamical Systems | Lecture 5

https://www.youtube.com/watch?v=nF9ZDf1THeE

INTRODUCTION

There is an alternative geometric perspective for flows on the line that comes from physics. In this lecture we present the geometric perspective of potentials. The terminology comes from potential energy in physics and allows one to think about a particle "moving" around on a potential energy surface. Here we describe the method and compare it with the previously analysed phase lines.

We can consider a particle in a potential $V(x)$

$$\dot{x} = f(x) = -\frac{dV(x)}{dx}$$

$$V(x) = -\int f(x)\,dx$$

The potential function $V(x)$ describes the energy landscape and we will see that a steady-state solution corresponds to a local minimum in the potential function.

The time rate of change of the potential is

$$\frac{d}{dt}\left[V(x(t))\right] = \frac{dV}{dx}\frac{dx}{dt} = \frac{dV}{dx}x = \frac{dV}{dx}f(x) \quad f(x) = -\frac{dV}{dx}$$

$$\frac{d}{dt}\left[V(x(t))\right] = -\left(\frac{dV}{dx}\right)^2 \leq 0$$

So, the potential of the particle aways decreases along a trajectory and the system evolves to the lowest possible local energy state which corresponds to a fixed point of the system.

SIMULATIONS

Example **ds25L5.py** $\dot{x} = x - x^3$

$\dot{x} = x - x^3$ initial condition $x(0) = x_0$

The steady-states are $x_{ss} = \pm 1$ and $x_{ss} = 0$

Steady-states $x_{ss} = \pm 1$ are **stable**

$$f(x) = x - x^3 \quad f'(x) = 1 - 3x^2 \quad f'(x_{ss} = \pm 1) = -2 < 0$$

$f'(x_{ss} = 0) = 1 > 0$ $x_{ss} = 0$ is **unstable**

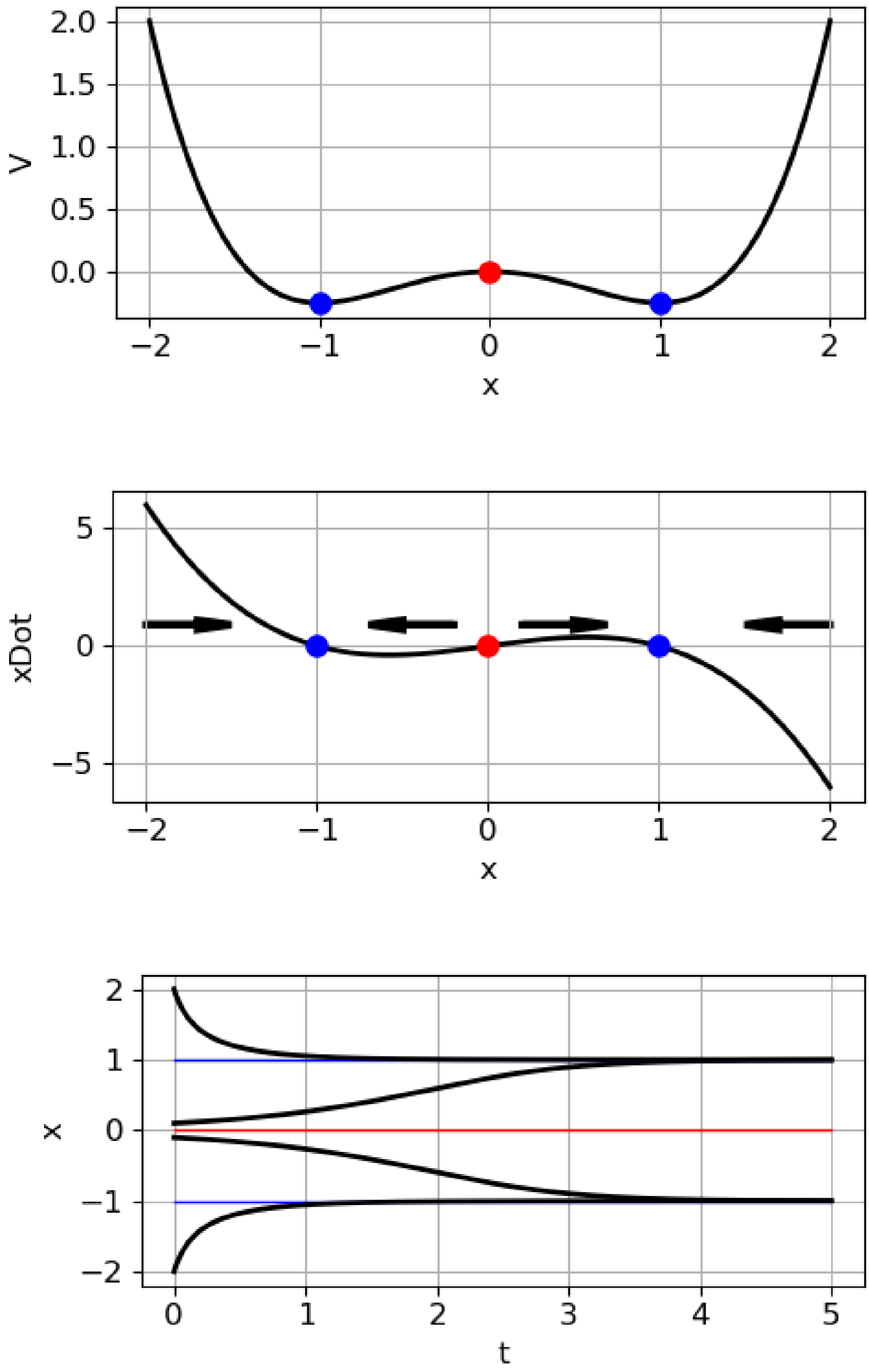
The potential function is

$$V(x) = -\int f(x)\,dx = -\int \left(x - x^3\right) dx = -\left(\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\right) + C$$

The constant C only moves the potential up or down and is of no significance, so we can set $C = 0$.

$$V(x) = -\left(\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\right)$$

As shown in the following graphs, the system evolves to the lowest local energy state at $x_{ss} = \pm 1$ from any initial condition except $x_{ss} = 0$.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