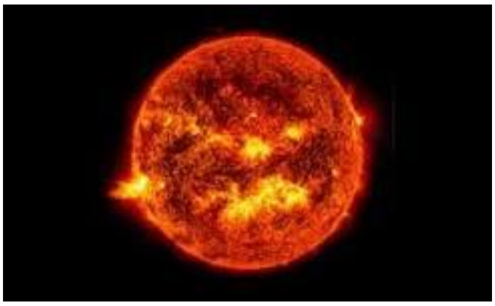
DOING PHYSICS WITH PYTHON

ENVIRONMENTAL AND CLIMATE PHYSICS

SOLAR RADIATION



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DOWNLOAD DIRECTORIES FOR PYTHON CODE

qmSun.py C001.py

Google drive

GitHub

SUN: BLACKBODY RADIATION

To a good approximation the Sun can be modelled as a **blackbody** to describe the electromagnetic radiation emitted from its surface. A summary of the relevant physical quantities is given in Table 1.

Table 1. List of physical quantities, units and values of constants used in the description of the radiation from a hot object.

Variable	Interpretation	Value / Unit
E	energy of photon	J
h	Planck's constant	6.62608×10^{-34} J.s
c	speed of electromagnetic radiation	$3.00\text{x}10^{8}$ m.s^{-1}
f	frequency of electromagnetic radiation	Hz
λ	wavelength of electromagnetic radiation	m
T	surface temperature of object	K
A	surface area of object	m^{2}
σ	Stefan-Boltzmann constant	5.6696×10^{-8} W.m^{-2}.K^{-4}
P	power emitted from hot object	W
ε	emissivity of object's surface	$0 \le \varepsilon \le 1$
R_λ	spectral exitance: power radiated per unit area per unit wavelength interval	(W.m^{-2}).m^{-1}
R_f	spectral exitance: power radiated per unit area per unit frequency interval	(W.m^{-2}).s^{-1}
k_B	Boltzmann constant	1.38066×10^{-23} J.K^{-1}

b_λ	Wien constant: wavelength	2.898×10^{-3} m.K
b_f	Wien constant: frequency	$b_f = 2.82 k_B / h$
λ_p	wavelength of peak in solar spectrum	5.0225×10^{-7} m
R_S	radius of the Sun	6.96×10^{8} m
R_E	radius of the Earth	6.96×10^{6} m
R_{SE}	Sun-Earth radius	6.96×10^{11} m
I_0	Solar constant	1.36×10^{3} W.m^{-2}
α	Albedo of Earth's surface	0.30
S_0	Solar constant	1360 W.m^{-2}

The wave nature of electromagnetic radiation is demonstrated by interference phenomena. However, electromagnetic radiation also has a particle nature. For example, to account for the observations of the radiation emitted from hot objects, it is necessary to use a particle model, where the radiation is considered to be a stream of particles called **photons**. The energy of a photon, E is

(1) $E = h\,f$

The electromagnetic energy emitted from an object's surface is called ***thermal radiation*** and is due to a decrease in the internal energy of the object. This radiation consists of a continuous spectrum of frequencies extending over a wide range. Objects at room temperature emit mainly infrared and it is not until the temperature reaches about 800 K and above those objects glows visibly.

A **blackbody** is an object that completely absorbs all electromagnetic radiation falling on its surface at any temperature. It can be thought of as a perfect absorber and emitter of radiation. The power emitted from a blackbody, P is given by the **Stefan-Boltzmann law** and it depends only on the surface area of the emitter, A and its surface temperature, T

(2) $$P = A\sigma T^4$$

A more general form of equation 2 is

(3) $$P = \varepsilon A\sigma T^4$$

where ε is the **emissivity** of the object. For a blackbody, $\varepsilon = 1$. When $\varepsilon < 1$ the object is called a **graybody** and the object is not a perfect emitter and absorber.

The amount of radiation emitted by a blackbody is given by **Planck's radiation law** and is expressed in terms of the **spectral exitance** for **wavelength** or **frequency** R_λ *or* R_f respectively

(4) $$R_\lambda = \frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp\left(\dfrac{hc}{k_B T \lambda}\right) - 1}$$ [W.m^{-2}.m^{-1}]

or

(5) $$R_f = \frac{2\pi h f^3}{c^2} \frac{1}{\exp\left(\dfrac{hf}{k_B T}\right) - 1}$$ [W.m^{-2}.s^{-1}]

The **spectral exitance** is also called the **spectral irradiance**. In the literature, many different terms and symbols are used for the spectral exitance. Sometimes the terms and the units given are wrong or misleading. *Note units for equation 1: The units in the denominator should be thought of in terms of* m^2.m *where the first factor* m^2 *corresponds to the surface area of the blackbody and the second unit of length* m *corresponds to the wavelength of the emitted light.*

The **power radiated per unit surface of a blackbody**, P_A/A within a wavelength interval or bandwidth, (λ_1, λ_2) or frequency interval or bandwidth (f_1, f_2) are given by equations 6 and 7

(6) $$P_A / A = \int_{\lambda_1}^{\lambda_2} R_\lambda \, d\lambda = \int_{\lambda_1}^{\lambda_2} \left(\frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{k_B T \lambda}\right) - 1} \right) d\lambda$$

(7) $$P_A / A = \int_{f1}^{f2} R_f \, df = \int_{f_1}^{f_2} \left(\frac{2\pi h f^3}{c^2} \frac{1}{\exp\left(\frac{h f}{k_b T}\right) - 1} \right) df$$

The equations 6 and 7 give the Stefan-Boltzmann law (equation 2) when the bandwidths extend from 0 to ∞.

Wien's Displacement law states that the wavelength λ_{peak} corresponding to the peak of the spectral exitance given by equation 4 is inversely proportional to the temperature of the blackbody and the frequency f_{peak} for the spectral exitance peak frequency given by equation 5 is proportional to the temperature

(8) $$\lambda_{peak} = \frac{b_\lambda}{T} \qquad f_{peak} = b_f \, T$$

The peaks in equations 4 and 5 occur in different parts of the electromagnetic spectrum and so

(9) $$f_{peak} \neq \frac{c}{\lambda_{peak}}$$

The Wien's Displacement law explains why long wave radiation dominates more and more in the spectrum of the radiation emitted by an object as its temperature is lowered.

When classical theories were used to derive an expression for the spectral exitances R_λ and R_f, the power emitted by a blackbody diverged to infinity as the wavelength became shorter and shorter. This is known as the **ultraviolet catastrophe**. In 1901 Max Planck proposed a new radical idea that was completely alien to classical notions, electromagnetic energy is **quantized**. Planck was able to derive the equations 4 and 5 for blackbody emission and these equations are in complete agreement with experimental measurements. The assumption that the energy of a system varies in a continuous manner. Energy can only exist in integer multiples of the lowest amount or quantum, $h\,f$.

This step marked the very beginning of modern quantum theory

Assuming the Sun is as a blackbody, we can calculate the following physical quantities using the Python Code **C001.py**:

Power radiated from the Sun

The intensity of the solar radiation at the top of the Earth's atmosphere is called the **solar constant** S_0. Hence, the power output of the Sun P_S is

$$P_S = S_0\left(4\pi\, R_{SE}{}^2\right) = 3.71\times10^{26}\ \text{W}$$

Intensity of radiation at Sun's surface

$$I_S = P_S\,/\left(4\pi\, R_S\, 2\right) = 6.09\times10^{7}\ \text{W.m}^{-2}$$

Photosphere temperature

$$T_S = \left(\frac{I_S}{\sigma}\right)^{1/4} = 5726\ \text{K}$$

Peak spectral wavelength

$$\lambda_{peak} = \frac{b_\lambda}{TS} = 510\ \text{nm} = 0.51\ \mu\text{m}$$

The blackbody spectrum the Sun is shown in figures 1 and 2 using the Python Code **qmSun.py**. The function **colour(wL)** sets the colour in the visible spectrum for each wavelength. A summary of the simulation results is displayed in the Console Window.

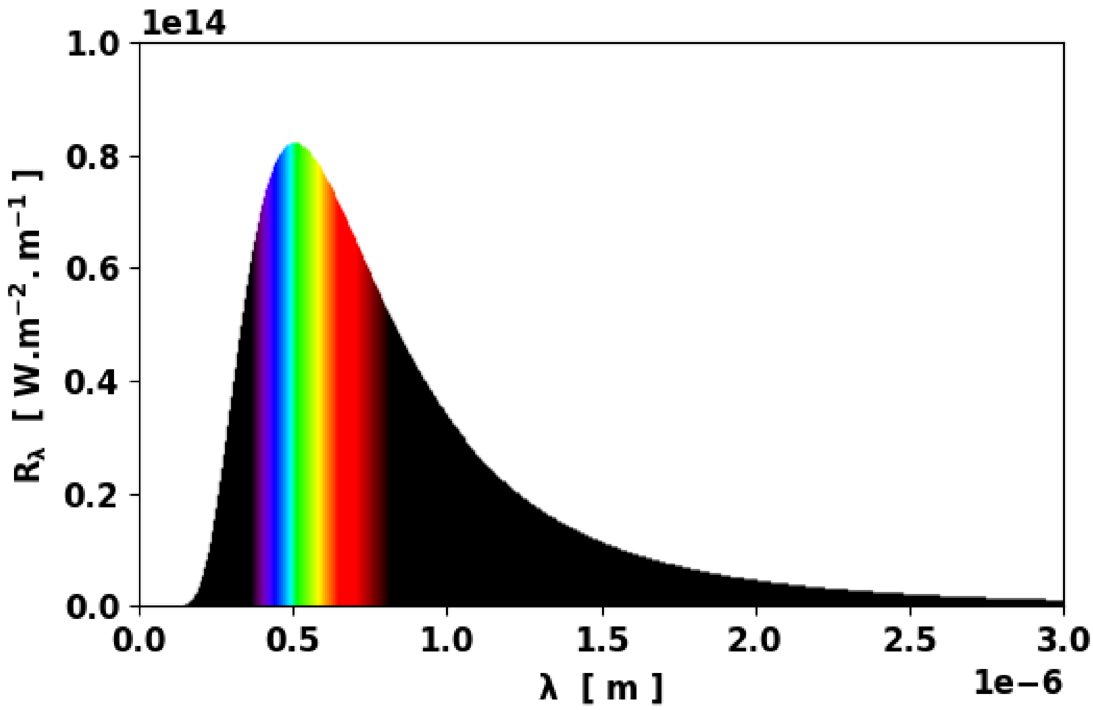

Fig. 1. Spectral exitance curve T = 5770 K

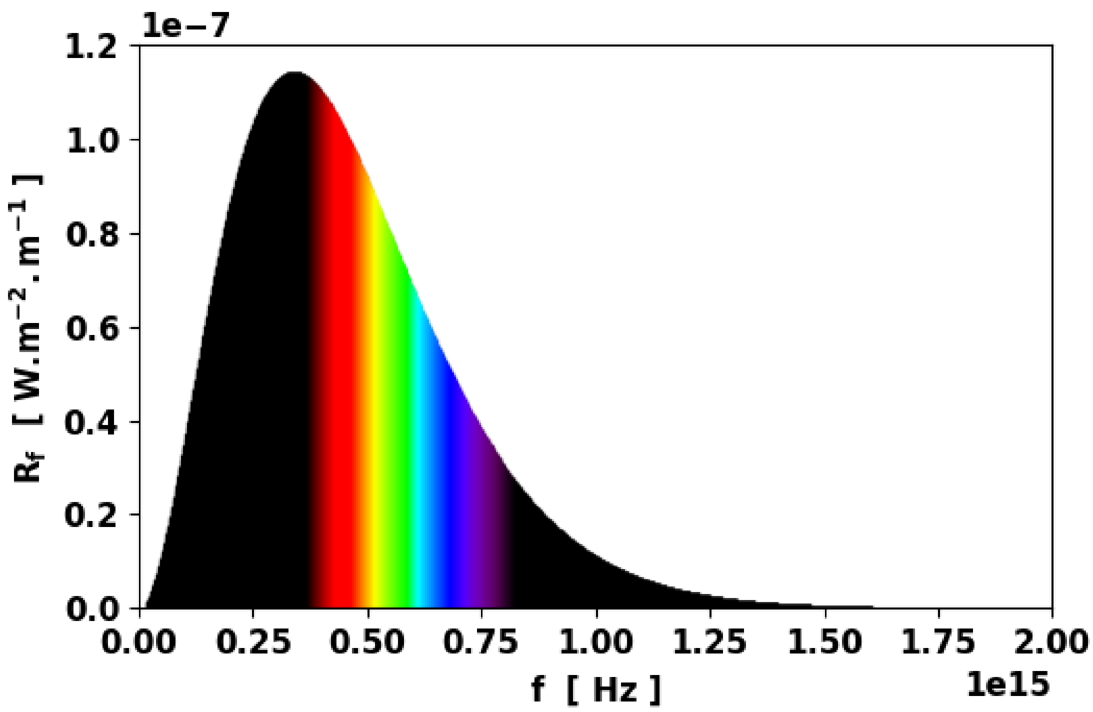

Fig. 2. Spectral exitance curve T = 5770

Console output (qmSun.py)

```
Sun: temperature of photosphere, T_S = 5770 K
Peak in Solar Spectrum
  Theory: Wavelength at peak in spectral exitance
              wL_peak = 5.02e-07 m
  Graph: Wavelength at peak in spectral exitance
              wL_peak = 5.04e-07 m
Theory: Frequency at peak in spectral exitance
            f_peak = 3.39e+14 Hz
  Graph:  Frequency at peak in spectral exitance
            f_peak = 3.40e+14 Hz
Total Solar Power Output
  P_Stefan_Boltzmann = 3.79e+26  W
  P_wL  = 3.77e+26  W       P_f      = 3.79e+26  W
IR / visible / UV
  P_IR  = 1.92e+26 W   percentage 50.95
  P_vis = 1.39e+26 W   percentage 36.82
  UV    = 4.61e+25 W   percentage 12.23
Sun - Earth
Theory: Solar constant I_O   = 1.360e+03  W/m^2
Computed: Solar constant I_E = 1.34e+03 w/m^2
Surface temperature of the Earth, T_E  = 254 K = -19 deg C
Execution time:  41.06
```

The calculations for the Console summary are described by the following text:

The total power output of the Sun P_S can be estimated by using the Stefan-Boltzmann law, equation 2, and by finding the area under the curves for R_λ and R_f using equations 6 and 7. From observations on the Sun, the peak in the electromagnetic radiation emitted has a wavelength, λ_{peak} = 502.25 nm (green). The temperature of the Sun's surface (photosphere) can be estimated from the Wien displacement law, equation 8.

The distance from the Sun to the Earth, R_{SE} can be used to estimate of the surface temperature of the Earth T_E if there was no atmosphere. The intensity of the Sun's radiation reaching the top of the atmosphere, S_0 is known as the **solar constant**

(10) $$S_0 = \frac{P_S}{4\pi R_{SE}^{\ 2}}$$

The power absorbed by the Earth, P_{Eabs} is

(11) $$P_{Eabs} = (1 - \alpha)\pi R_E^{\ 2} I_0$$

where α is the albedo (the reflectivity of the Earth's surface).

Assuming the Earth behaves as a blackbody then the power of the radiation emitted from the Earth, P_{Erad} is

(12) $$P_{Erad} = 4\pi R_E^{\,2} \sigma T_E^{\,4}$$

It is known that the Earth's surface temperature has remained relatively constant over many centuries, so that the power absorbed and the power emitted are equal, so the Earth's equilibrium temperature T_E is

(13) $$T_E = \left(\frac{(1-\alpha) I_0}{4\sigma} \right)^{0.25}$$