DOING PHYSICS WITH PYTHON

COMPUTATIONAL OPTICS

MICHELSON INTERFEROMETER

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DOWNLOAD DIRECTORIES FOR PYTHON CODE

Google drive

GitHub

emMichelsonA.py

Plane waves: uniform screen intensity plots; animation of screen intensity
Plant growth calculation

emMichelsonA1.py

Plane waves: uniform screen intensity plot

emMichelsonB.py

Point source: screen intensity plots; animation of screen intensity

emMichelsonC.py

Point sources: screen intensity plot [2D] circular fringe pattern

emMichelsonD.py

Plane waves: uniform screen intensity plots with M2 mirror tilted

INTRODUCTION

Interferometers are basic optical tools used to precisely measure wavelength, distance, index of refraction, and temporal coherence of optical beams.

In the Michelson interferometer, light from a source is split into two beams at a beam splitter (partially reflecting mirror). One beam travels to a fixed mirror M_1 and is reflected back to the beam splitter while the other beam is reflected from a movable mirror M_2 back to the beam splitter. The two beams recombine and are then detected as shown in figure 1. The two beams must be mutually coherent for interference fringes to be observed. This process is known as interference by division of amplitude. It is assumed that the beam splitter divides the two beams equally and is oriented at 45^o to the source beam.

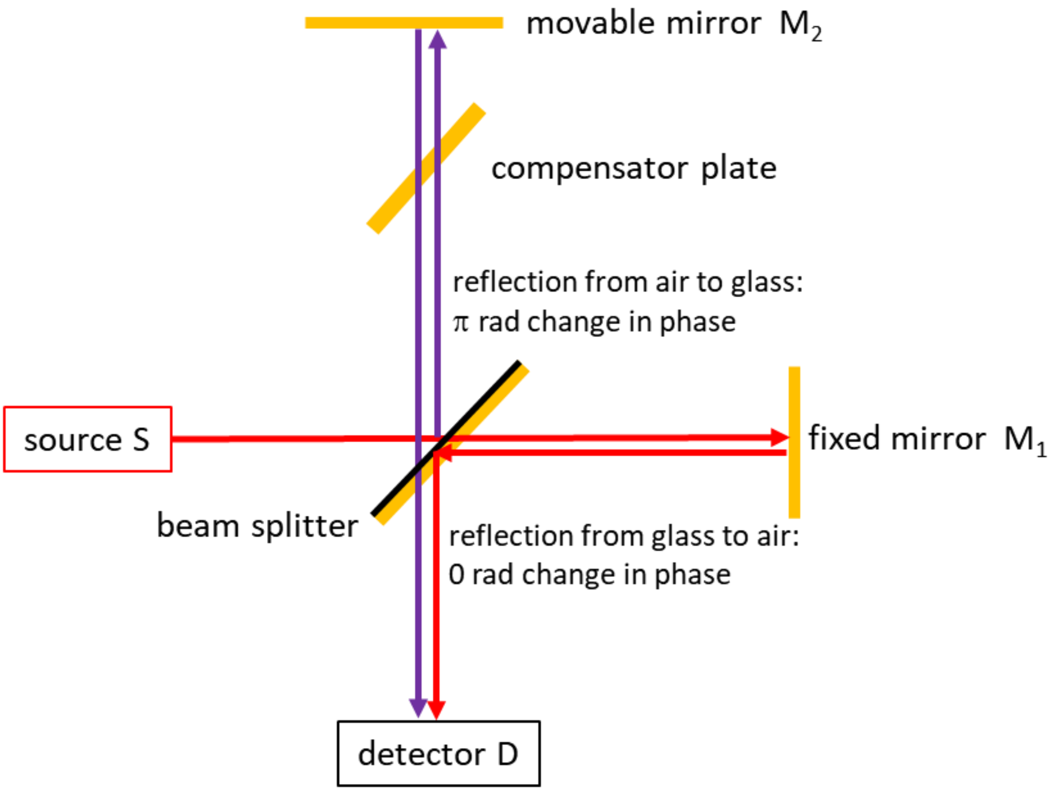

Fig. 1. Michelson interferometer.

Note, there is a π change in phase at the beam splitter for the beam reflected from mirror M_2. So, there is a π difference in the phase of the two light beams when they combine at the detector.

A **linear optical equivalent** of the Michelson interferometer helps to understand the optical path differences and aids in calculating the detector screen intensity. The mirror M_1 and the source S are replaced with virtual images M_{1I} and S_I (figure 2). The detected interference pattern is the same in both configurations where the separation distance between the two mirrors is given by $\Delta d = |d_2 - d_1|$.

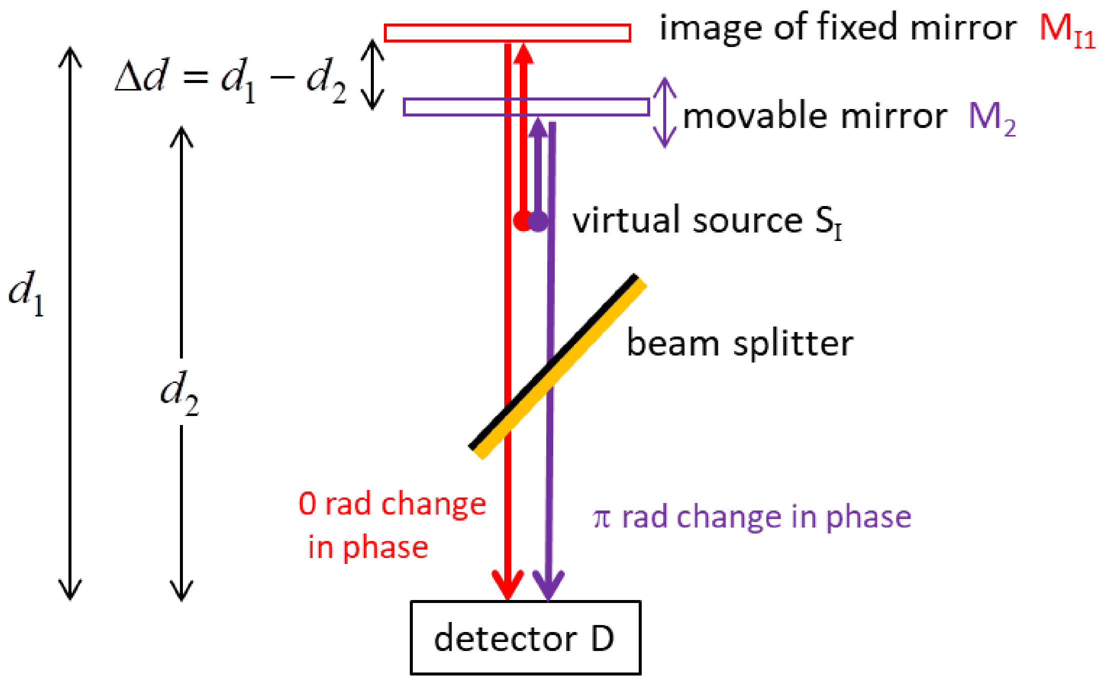

Fig. 2. Linear optical equivalent arrangement of the Michelson interferometer.

For the rays that travels along the central path of the interferometer, the optical path difference between the light reflected from the two mirrors M_{1I} and M_2 is $|2\Delta d|$. Hence, at the centre of the detector

Dark spot (destructive interference)

$$|2\Delta d| = n\lambda$$

Bright spot (constructive interference)

$$|2\Delta d| = \left(n + \tfrac{1}{2}\right)n\lambda$$

where $n = 0, 1, 2, 3, \ldots$ and remember that the two beams are π out of phase at the detector.

Monochromatic parallel plane wave interference

The intensity of the detector screen is due to the interference of the two plane waves. The wavefronts from the reflection from mirror M_{1I} are parallel to the screen, while the wavefronts from the reflection from mirror M_2 are tilted at an angle to the screen. The angle $\theta / 2$ is the tilt of mirror M_2 w.r.t. the X axis and θ is the angle of the plane wave from mirror M_2 w.r.t. to the Z axis as shown in figure 3.

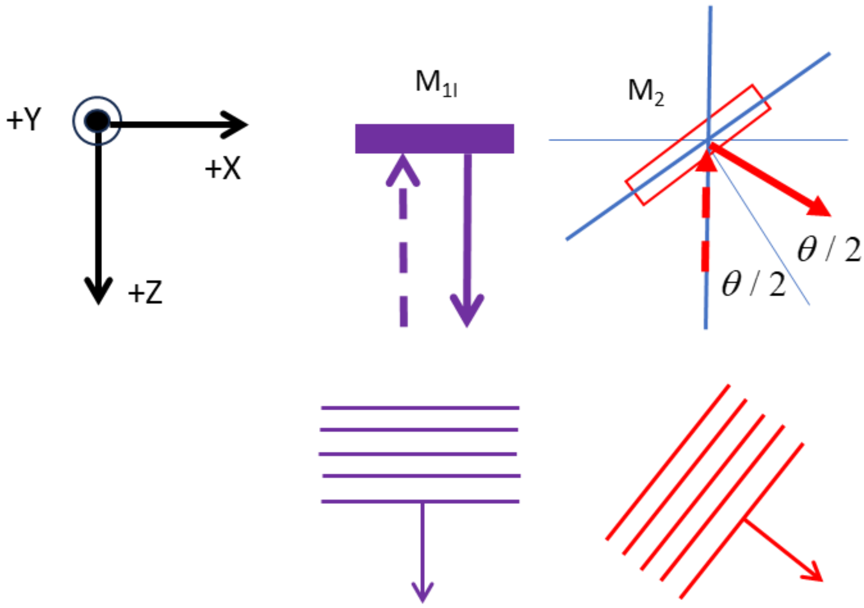

Fig. 3. Mirror geometry for a tilted movable mirror M_2. The mirror M_2 is tilted through the angle $\theta / 2$ w.r.t. the Z axis, while the beam is deflected through an angle θ.

Point on mirror M_{1I} $\quad (x_1, y_1, z_1 = 0)$

Point on mirror M_2 $\quad (x_2, y_2, z_2 = d)$

Point on detector $\quad (x_D, y_D, z_D)$

The equation of a plane monochromatic wave can be expressed as

$$\vec{E} = \vec{E}_0 \exp\left(i\vec{k} \bullet \vec{r} + \phi\right)$$

For the configuration shown in figures 2 and 3, for plane monochromatic beams of wavelength λ the time independent electric fields of the two beams the point at the point $\left(x_D, z_D\right)$ on the screen of the detector are

(1A) $E_{D1} = E_0 \, \mathrm{e}^{i\left(k\left(z_D + 2\Delta d\right) + i\pi\right)}$

(1B) $E_{D2} = E_0 \, \mathrm{e}^{i\left(k\left(z_D \cos\theta + x_D \sin\theta\right)\right)}$

Then, the resultant electric field E_D at the detector is the superposition of the fields E_{D1} and E_{D2}.

(1C) $E_D = E_{D1} + E_{D2}$

Therefore, the intensity S_D of the combined beam at the detector screen is

(1D) $S_D = E_D^{\,*} E_D$

More generally, for tilts is both the X and Y direction, produces a relative phase variation of

$$\phi = k_x \, x \sin\left(\theta_x\right) + k_x \, y \sin\left(\theta_y\right)$$

where $\theta_x / 2$ represents the tilt of the mirror in the X dimension

and $\theta_y / 2$ represents the amount of tilt in the Y dimension.

Two mirrors are precisely parallel with no tilt $(\theta = 0)$

If the **two mirrors are precisely parallel with no tilt** $(\theta = 0)$ as shown in figures 2, 3 and 4, then the detector screen is uniformly illuminated over its whole area. The intensity on the detector screen by can be calculated from the superposition of the electric fields propagated from the two mirrors. The geometry is shown in figure 4.

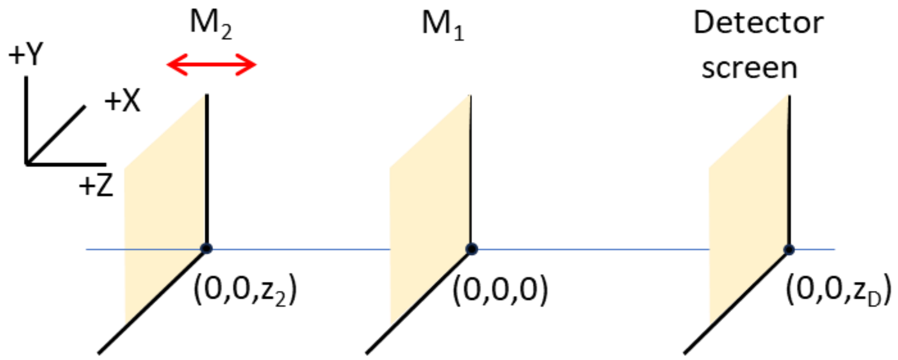

Fig. 4. Geometry of a Michelson interferometer for calculating the detector screen intensity from the electric fields propagated from each mirror.

The screen will be dark when the difference in the optical path lengths of the two beams is an integral multiple of a wavelength and bright when the there is an odd multiple of a half-wavelength (figure 4 **emMichelsonA.py**).

Dark (destructive interference) $2\Delta d = n\lambda$

Bright (constructive interference) $2\Delta d = (n + 1/2)\lambda$

Regions of uniform phase, called **fringe**s (in this case individual stripes), have the same intensity. As the delay z_2 is varied, the fringes seem to 'move' across the detector and the entire beam 'blinks' on and off as the delay path z_2 is varied.

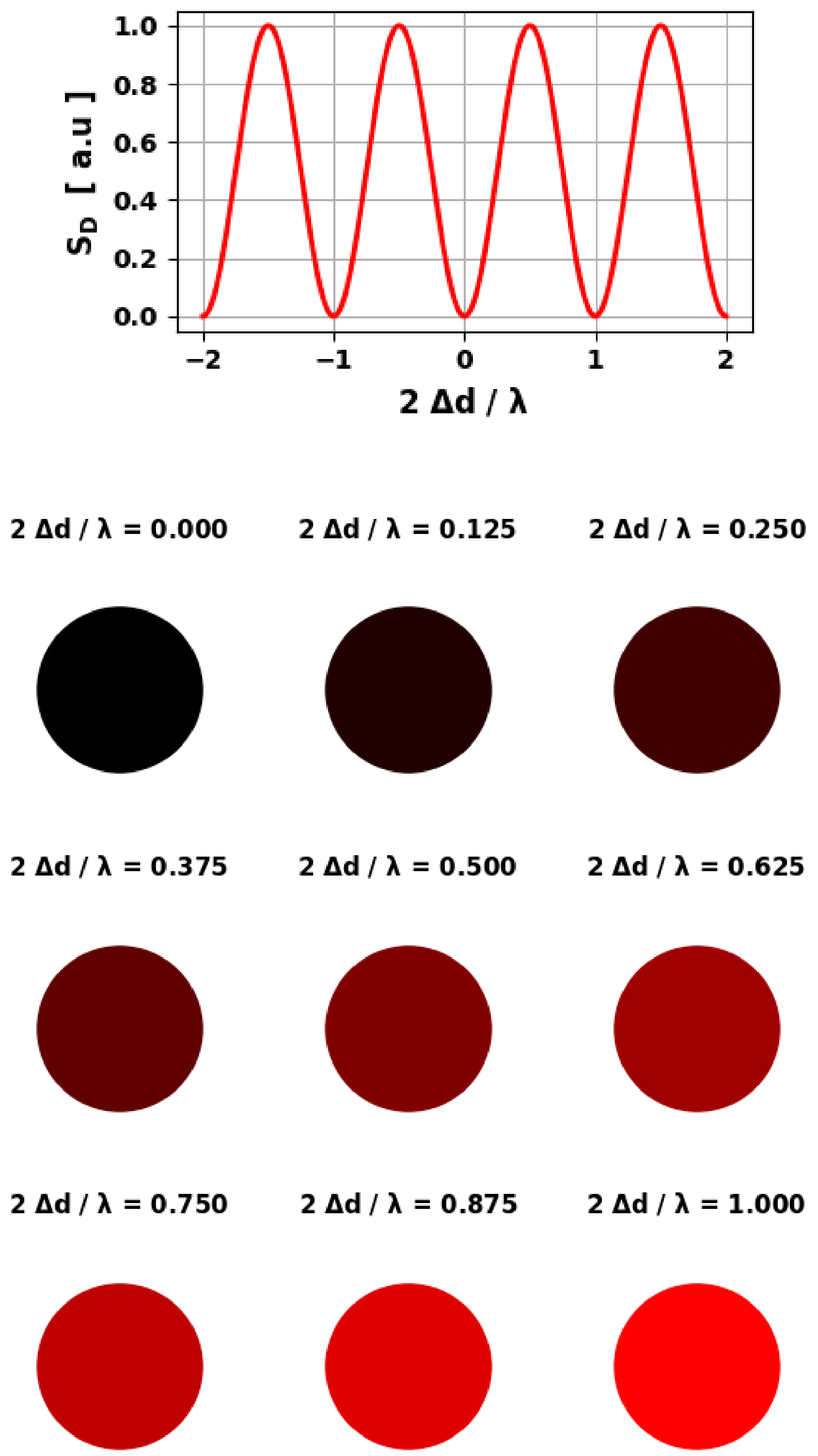

Fig. 4. The detector screen S_D intensity as a function of the optical path length difference for the reflections from the two mirrors.

VIEW ANIMATION

The animation shows the variation in the detector screen intensity as the position of mirror M_2 changes. The number n_f of fringes increases by one each time the distance of the mirror M_2 is moved through a distance is given by $2\Delta d / \lambda$.

Example Plant Growth

The Michelson interferometer can be used to measure small displacement accurately. For example, it is possible to measure the growth rate of a plant. The plant was attached to the movable mirror M_2 and its position was adjusted to give a black field of view on the detector screen. A helium-neon laser was used with a wavelength of 632.8 nm. As the plant grew, the distance between the mirrors increased. So, the fringe pattern changed in a manner as observed in the animation. In an 8.0 hour period, (3415 ± 5) dark fringes crossed the field of view. Estimate the growth rate of the plant in mm.h^{-1} and its uncertainty.

emMichelsona.py

```
#%% Plant Growth Calculation
wL = 632.8e-9;      # wavelength [m]
dt = 8.3            # Time interval  [h]
nf = 3420;          # Number of fringes
dP = (nf * wL /2) * 1e3;   # distance moved by plant on mirror 2 [mm]
dPdt = dP/dt;       # rate of growth  [mm/h]
print('Inputs  ')
print('   wavelength = %3.1e  m ' % wL)
print('   time interval = %3.1f  h' % dt)
print('   fringes  = %3.0f ' %nf)
print('Outputs  ')
```

```
print('   growth distance = %3.5f  mm' % dP)
print('   rate of growth  = %3.5f  mm/h ' % dPdt)
```

```
Inputs
   wavelength = 6.3e-07  m
   time interval = 8.0  h
   fringes  = 3410
Outputs
   growth distance = 1.07892  mm
   rate of growth  = 0.13487  mm/h
Inputs
   fringes  = 3415
Outputs
   growth distance = 1.08051  mm
   rate of growth  = 0.13506  mm/h
Inputs
   fringes  = 3420
Outputs
   growth distance = 1.08209  mm
   rate of growth  = 0.13526  mm/h
```

Growth rate = $\left(0.1351 \pm 0.0002\right)$ mm.h^{-1}

emMichelsona.py

```
#%% Plant Growth Calculation
wL = 632.8e-9;       # wavelength [m]
dt = 8.3             # Time interval  [h]
nf = 3420;           # Number of fringes
dP = (nf * wL /2) * 1e3;   # distance moved by plant on mirror 2 [mm]
dPdt = dP/dt;        # rate of growth  [mm/h]

print('Inputs  ')
print('   wavelength = %3.1e  m ' % wL)
print('   time interval = %3.1f  h' % dt)
print('   fringes  = %3.0f ' %nf)
print('Outputs  ')
print('   growth distance = %3.5f  mm' % dP)
print('   rate of growth  = %3.5f  mm/h ' % dPdt)You
```

Since we have plane wave illumination the intensity distribution is uniform on the screen as shown in figure 4. Figure 5 shows the detector screen intensity variation as the position of the mirror M_2 is changed (z_2) for three different wavelengths.

From equations 1 after lots pf algebra, it can be shown that if the two mirrors are precisely parallel $(\theta = 0)$, then the intensity on the screen is

$$S_D = S_0 \sin^2\left(k\,\Delta d\right)$$

and the whole area of the screen will be uniformly illuminated (figure 5).

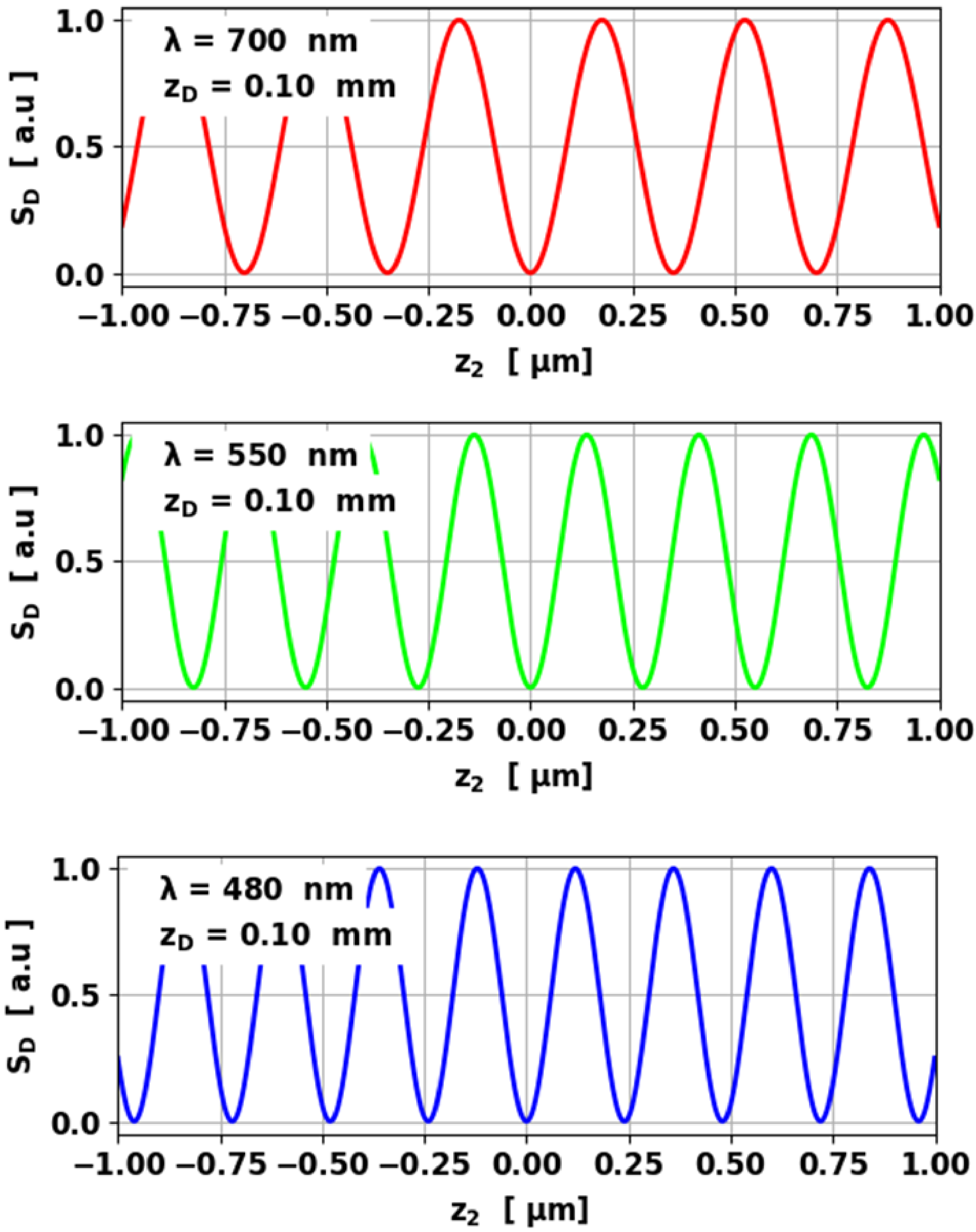

Fig. 5. Screen intensity as a function of the position of mirror M_2. The separation between fringes (dark screen or bright screen) is equal to a half wavelength change in the position of mirror M_2. The shorter the wavelength then the small distance mirror M_2 moves between fringes. Thus, the Michelson interferometer can be used to make accurate wavelength measurements. **emMichelsonA1.py**

When two waves combine constructively on the detector screen, a bright image (fringe) is observed. When mirror M_2 moves through a distance $\lambda / 2$ another bright image is formed since the optical path length of beam 2 increases by one wavelength λ. Consequently, by counting the number of fringes m passing a given point as M_2 is moved a distance Δd, an observer can measure minute displacements that are accurate to a fraction of a wavelength, as shown by the relationship

$$\Delta d = m\left(\frac{\lambda}{2}\right)$$

Example

A sodium lamp is used with a Michelson interferometer. Sodium has two yellow spectral lines with very similar wavelengths of 589.0 and 589.6 nm. The spectral line 589.0 nm was observed and mirror M_2 was moved through a distance 7068 nm. How many fringes would be observed?

$$\Delta d = m\left(\frac{\lambda}{2}\right) \quad m = \frac{2\Delta d}{\lambda} = \frac{(2)(7068)}{589} = 24$$

For the same number of fringes, what distance would mirror M_2 move for the 589.6 nm sodium line?

$$\Delta d = m\left(\frac{\lambda}{2}\right) = \frac{(24)(589.6)}{2}\ \text{nm} = 7075.2\ \text{nm}$$

For the longer wavelength of, 589.6 nm, mirror M_2 would have to move a distance 7.2 nm greater than for the 589.0 nm spectral line.

Mirror M2 rotated about the Y axis (waves reflected by M2 at an angle θ to the Z axis)

The intensity of the detector screen is due to the interference of the two plane waves. The wavefronts from the reflection from mirror M_{1I} are parallel to the screen, while the wavefronts from the reflection from mirror M_2 are tilted at an angle to the screen (figure 3). The resultant interference pattern shows a series of vertical bright and dark equally spaced fringes. The angle $\theta / 2$ is the tilt of mirror M_2 w.r.t. the X axis and θ is the angle of the plane wave from mirror M_2 w.r.t. to the Z axis as shown in figure 3. That is, by the law of reflection, the beam returning from the misaligned mirror deviates from the 'ideal' path by an angle θ.

Assume that there are bright fringes at screen position x_D and the adjacent fringe at $\left(x_D + dx_D\right)$. Then the phase difference between two fringes must be 2π. So, from equation 1B

$$k\left(z_D \cos\theta + x_D \sin\theta - 2\pi\right) = k\left(z_D \cos\theta + \left(x_D + dx_D\right)\sin\theta\right)$$

Hence, the fringe spacing is

$$dx_D = \frac{\lambda}{\sin\theta}$$

When the tilt angle goes to zero and the two mirrors become aligned, then

$$\theta \to 0 \quad \sin\theta \to 0 \quad \Delta x_D \to \infty$$

Thus, the fringes disappear and the screen becomes uniformly illuminated as described above.

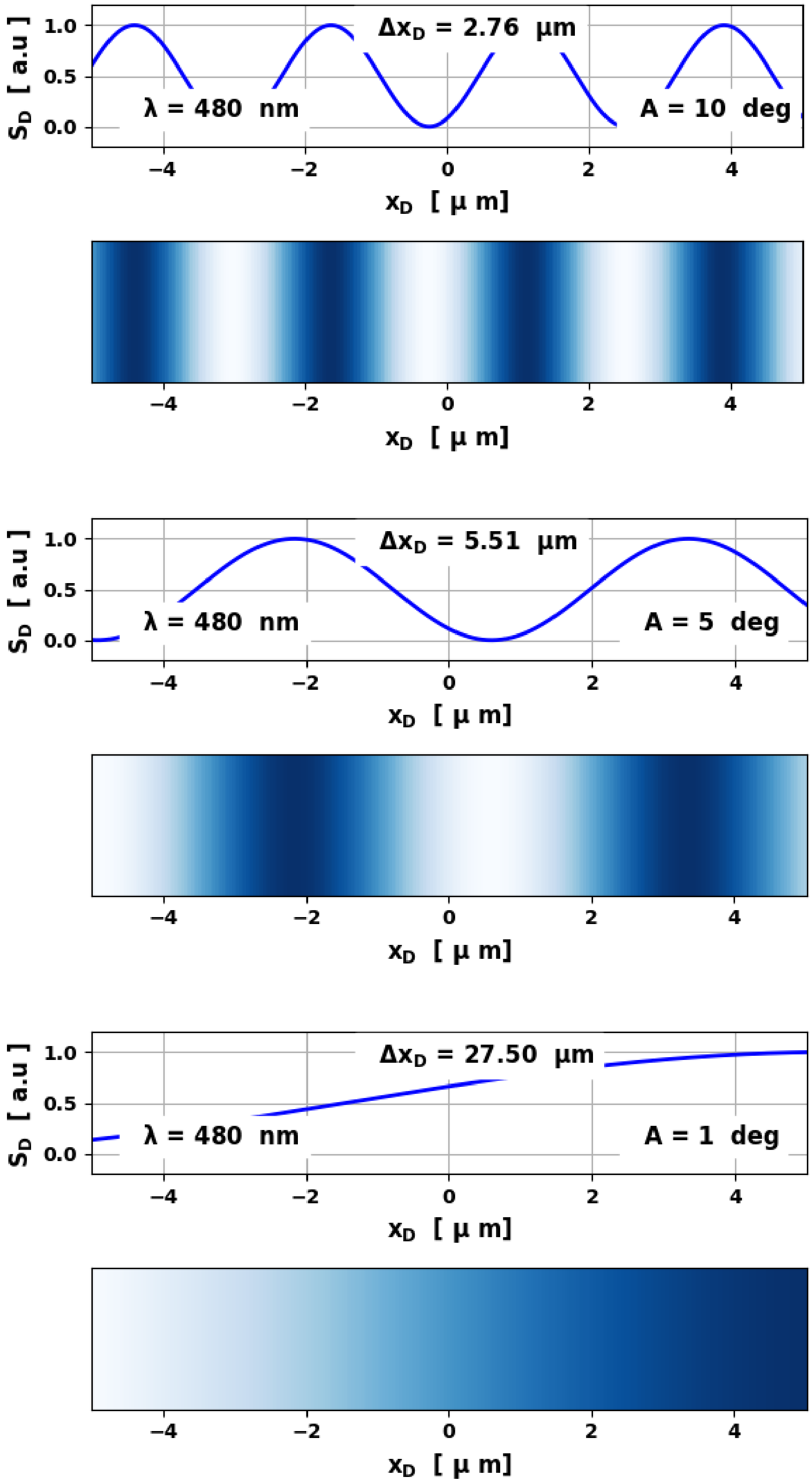

Fig. 6. When the tilt angle decreases, the fringe separation becomes smaller. $\theta \to 0 \quad \sin\theta \to 0 \quad \Delta x_D \to \infty$ **emMichelsonD.py**

The fringe separation becomes smaller as the wavelength decreases and when mirror M_2 is moved, the fringes move in a horizontal direction across the detector screen (figure 7.

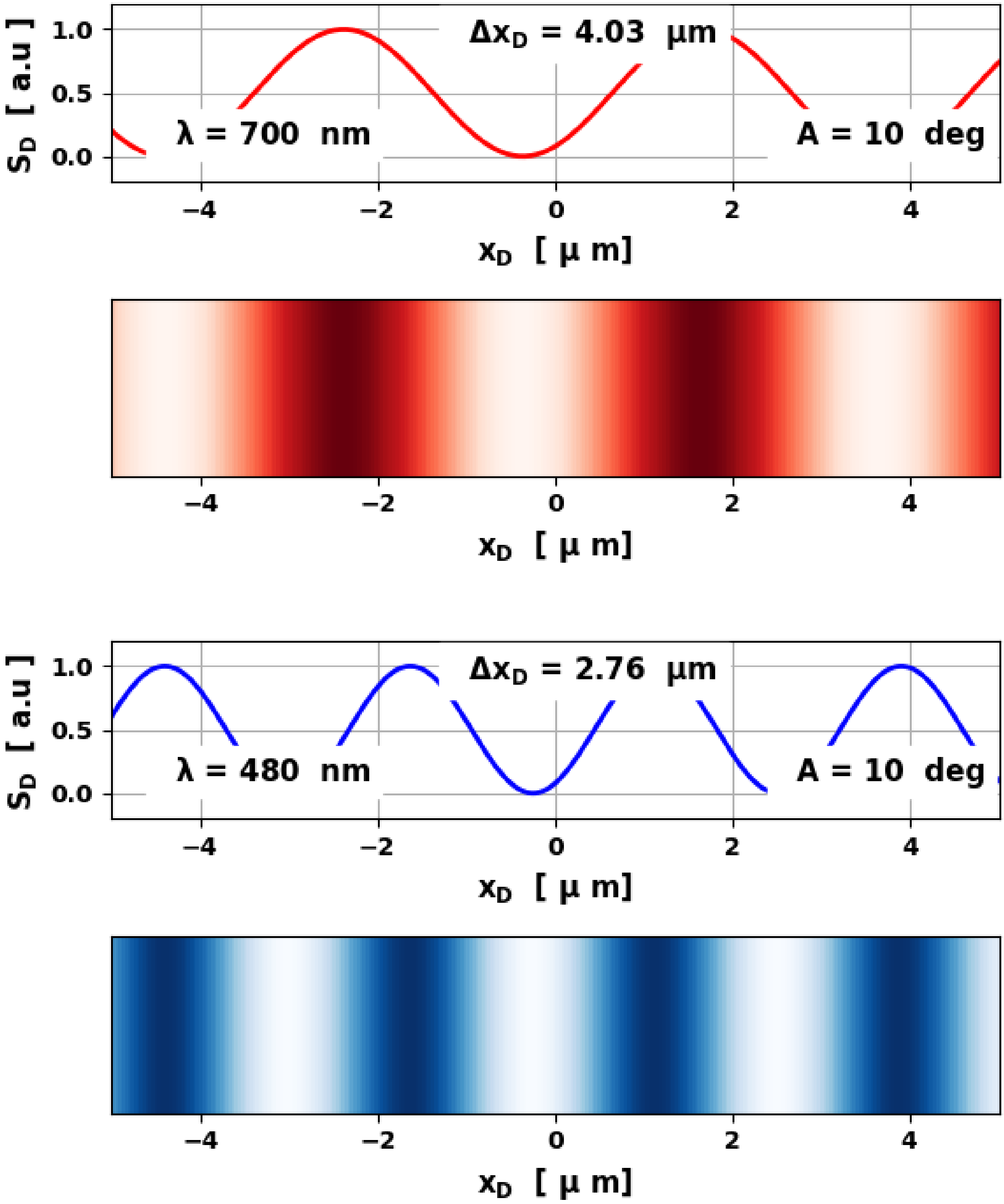

Fig. 7. When M_2 is tilted, the fringe separation becomes smaller as the wavelength decreases and the fringes are of equal thickness.

emMichelsonD.py

Illumination by a monochromatic point source

You can calculate the fringe pattern for point source illumination The input parameters are the wavelength, the distance between the virtual sources and the maximum viewing angle. The spherical waves from the two-point sources produce circular fringes on a detector screen.

A dark spot is located at the centre of the fringe pattern if the distance between the virtual source points is

$$\Delta d = m\lambda \quad m = 0, 1, 2, \ldots$$

A bright spot is located at the centre of the fringe pattern if the distance between the virtual source points is

$$\Delta d = (m + 1/2)\lambda \quad m = 0, 1, 2, \ldots$$

m is called the order of the fringe.

Link to animation below which shows how the circular fringe pattern changes as the position of mirror M_2 changes with separation distance z_2.

VIEW ANIMATION

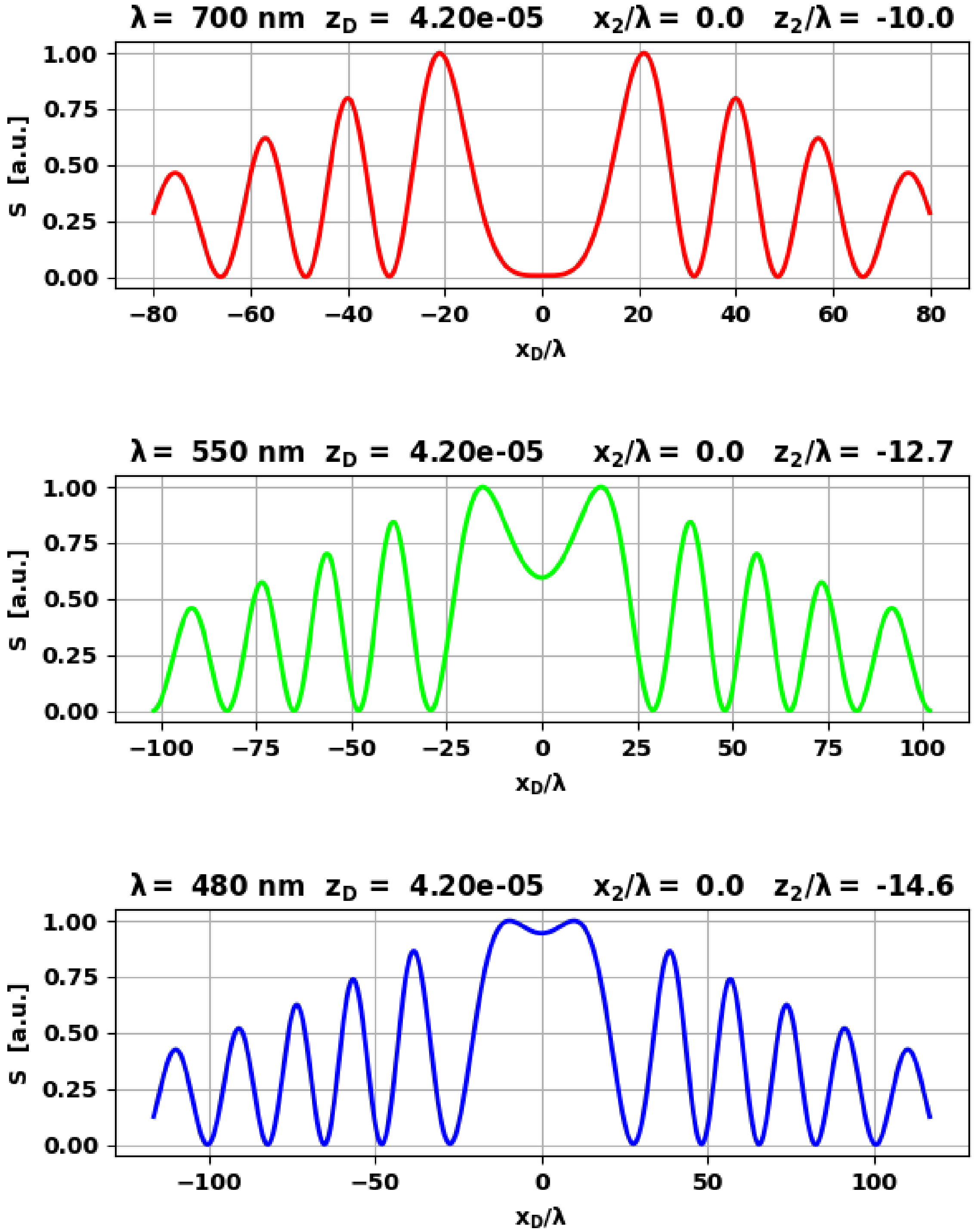

Fig. 8. Fringe pattern in a radial direction for the detector screen intensity. The smaller the wavelength, the greater the density of the bright and dark fringes. **emMichelsonB.py**

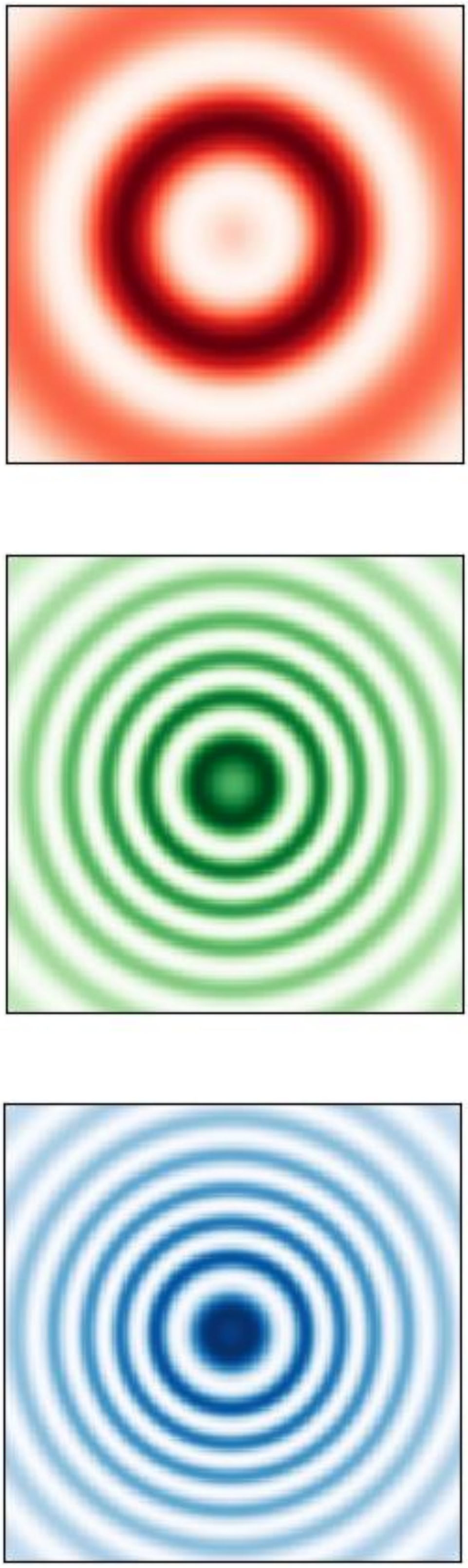

Fig. 9. Circular fringe pattern from two point sources separated along the Z axis. **emMichelsonC.py**

If the beam from the point source of mirror M2 is shifted off the XZ plane then the circular fringe pattern is distorted as shown in figure 10.

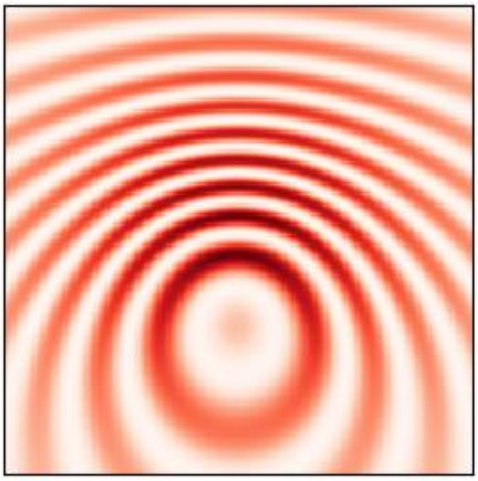

Fig. 10. Distorted fringe pattern for the point source of mirror M_2 is shifted off the XZ plane $\left(\lambda = 700\ \text{nm} \quad y_2 = 5\lambda\right)$.

Refractive Index of a Gas

A helium-neon laser $(\lambda = 632.8 \text{ nm})$ is used to measure the refractive index of a gas. In one arm of the Michelson interferometer, a glass chamber is placed with attachments for either evacuating or evacuating a gas. Consider a chamber of length 20 mm in length, initially empty. As a gas is slowly let into the chamber, you observe that dark fringes move past a reference line in the field of observation.

VIEW ANIMATION

When the chamber is filled to the desired pressure, (120 ± 5) fringes moved past the reference line. Estimate the refractive index *n* of the gas and its uncertainly.

Solution

$\lambda = 632.8\times10^{-9} \text{ m} \quad m = (120 \pm 5) \quad L = 2.0\times10^{-3} \text{ m} \quad n = ?$

The ray travels a distance L through glass chamber and another distance L upon reflection. Therefore, the total travel distance is $2L$.

When empty, the number of wavelengths that fit in this chamber is

$$N_1 = \frac{2L}{\lambda_1} \quad \lambda_1 = 632.8\times10^{-9} \text{ m}$$

When the chamber is fully filled with the gas, the number of wavelengths that fit in this chamber is

$$N_2 = \frac{2L}{\lambda_2} \quad \lambda_2 = \frac{\lambda_1}{n}$$

The number of fringes *m* observed when the chamber goes from empty to fill is simply the difference in the number of wavelengths fitting the chamber when empty compared to full

$$m = N_2 - N_1 = \frac{2L}{\lambda_1} n - \frac{2L}{\lambda_1} = \frac{2L}{\lambda_1}(n-1)$$

$$n = 1 + \frac{\lambda_1}{2L} m$$

The calculation can be done in the Console Window

```
wL = 632.8e-9

L = 20e-3

m = array([115,120,125])

n = 1+(2*L/wL)*m

➔ array([1.0018193, 1.0018984, 1.0019775])
```

$$n = (1.0019 \pm 0.0001)$$