DOING PHYSICS WITH PYTHON

COMPUTATIONAL OPTICS

RAYLEIGH-SOMMERFELD 1

DIFFRACTION INTEGRAL:

GAUSSIAN BEAM PROPAGATION

FROM A CIRCULAR APERTURE

Ian Cooper

Please email me any corrections, comments, suggestions or additions: **matlabvisualphysics@gmail.com**

DOWNLOAD DIRECTORIES FOR PYTHON CODE

Google drive

GitHub

emRSGB01.py Irradiance: planes

emRSGB01Z.py Irradiance: optical axis

Reference: Gaussian Beams

INTRODUCTION

The **Rayleigh-Sommerfeld diffraction integral of the first kind** is used to calculate the intensity of a **Gaussian beam** diffracted by a circular aperture.

The geometry of the aperture and observation spaces is shown in figure 1 and figure 2 shows an outline of how to the RS1 diffraction integral is computed in Python.

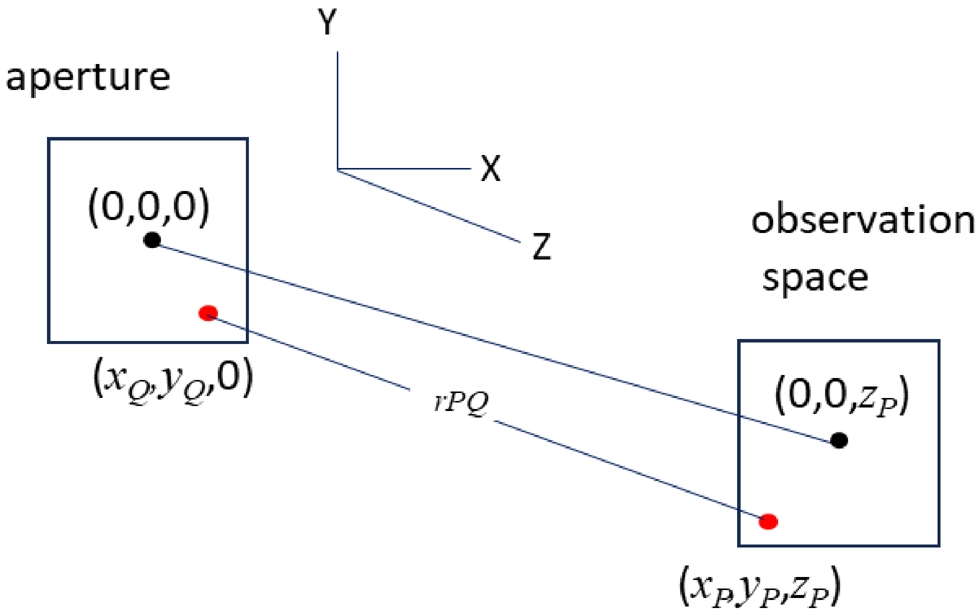


Fig. 1. Geometry of the aperture and observation spaces.

Rayleigh-Sommerfeld diffraction integral

$$E_P = \frac{1}{2\pi} \iint_{S_\mathrm{A}} E_Q \frac{e^{j k r_{PQ}}}{r_{PQ}{}^3} z_p (j k r_{PQ} - 1)\, \mathrm{d}S$$

numerical integration:
[2D] Simpson's 1/3 rule

$$E_P(x_P, y_P, z_P) = z_P \sum_{m=1}^{n_Q} \sum_{n=1}^{n_Q} \left(\left(\frac{e^{j k r_{PQmn}}}{r_{PQmn}{}^3} \right) \left(j k r_{PQmn} - 1 \right) \left(E_{Qmn} S_{mn} \right) \right)$$

nPxnP matrix **EP**

nQxnQ matrix **MP**

nQxnQ matrix **EQ**

nQxnQ matrix **S**

Fig. 2. Matrices used in computed the diffraction integral.

The electric field E_Q within the circular aperture of radius a has a Gaussian [2D] profile.

$$E_Q\left(x_Q, y_Q\right) = \exp\left(-\frac{x_Q 2 + y_Q{}^2}{2 s^2} \right)$$

where s^2 is the variance of the Gaussian [2D] profile. Figure 3 shows a [3D] view of the Gaussian beam. In the far-field, all XY planes have a Gaussian profile for the irradiance. As the the z distance increases from the centre of the aperture, the beam spreads and the peak irradiance decreases.

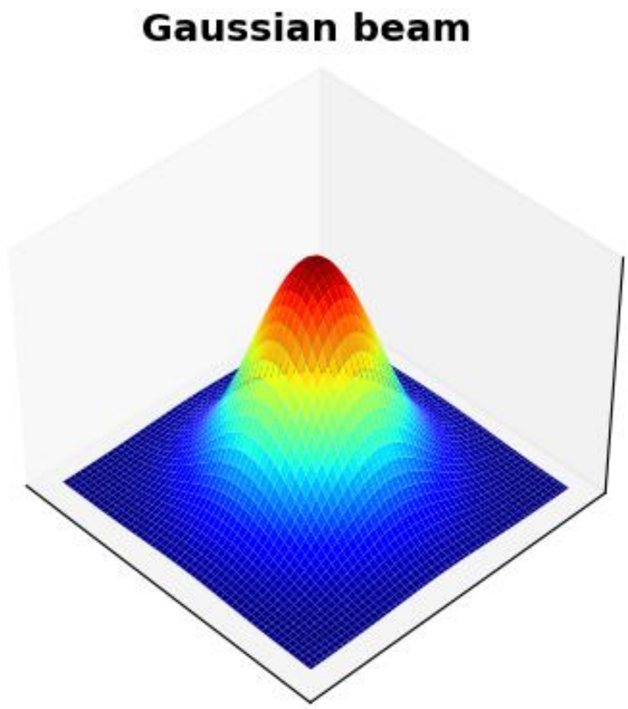


Fig. 3. [3D] view for the irradiance in an XY plane for the Gaussian beam. **emRSGB01.py**

For our Gaussian beam

$w(z)$ **beam spot** and w_0 is the **beam waist**

$w(z)$ is the radius of the beam at position z at which the irradiance is $1/e^2$ of its axial value. At position z along the axis, the beam spot $w(z)$ is given by

(1) $$w(z) = w_0 \sqrt{1 + \left(\frac{\lambda z}{\pi w_0^{\ 2}} \right)^2} \qquad w(0) = w_0$$

Also, the beam spot is calculated numerically in the Python Code **emRSGB01.py**

SIMULATIONS

In running the Python code **emRSGB01.py** a summary of the input and output parameters are displayed in the Console Window.

```
nQ = 199   nP = 237
aperture radius = 0.400  mm
Gaussian beam: width s = 0.200  mm
Wavelength wL = 632.8  nm
Observation space: zP = 1.500  mm
Gaussian beam waist w0/a = 0.707
Gaussian beam spot (theoretical) wT/a = 2.763
Relative max irradiance (normalized to 1 at zP = 1.0 m) = 62.550
 Execution time 165  s
```

The results of each simulation are displayed a set of Figure Windows. The irradiance **IXY** is normalized such that the peak value of the irradiance for **zP** = 1.00m is set to 1.

The Aperture Space

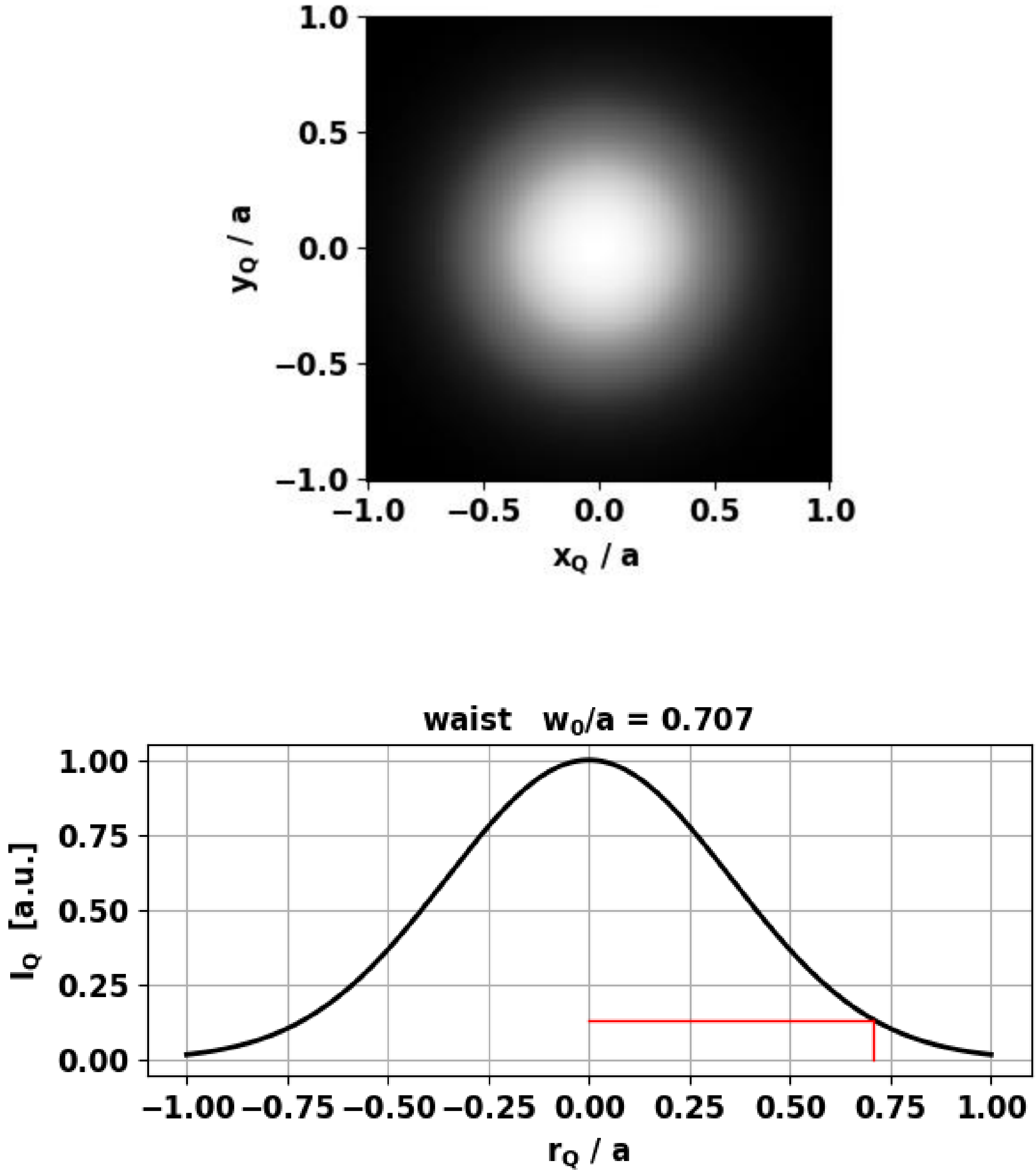


Fig. 4. The irradiance distribution for the circular aperture space of radius a. **emRSGB01.py**

Propagation space

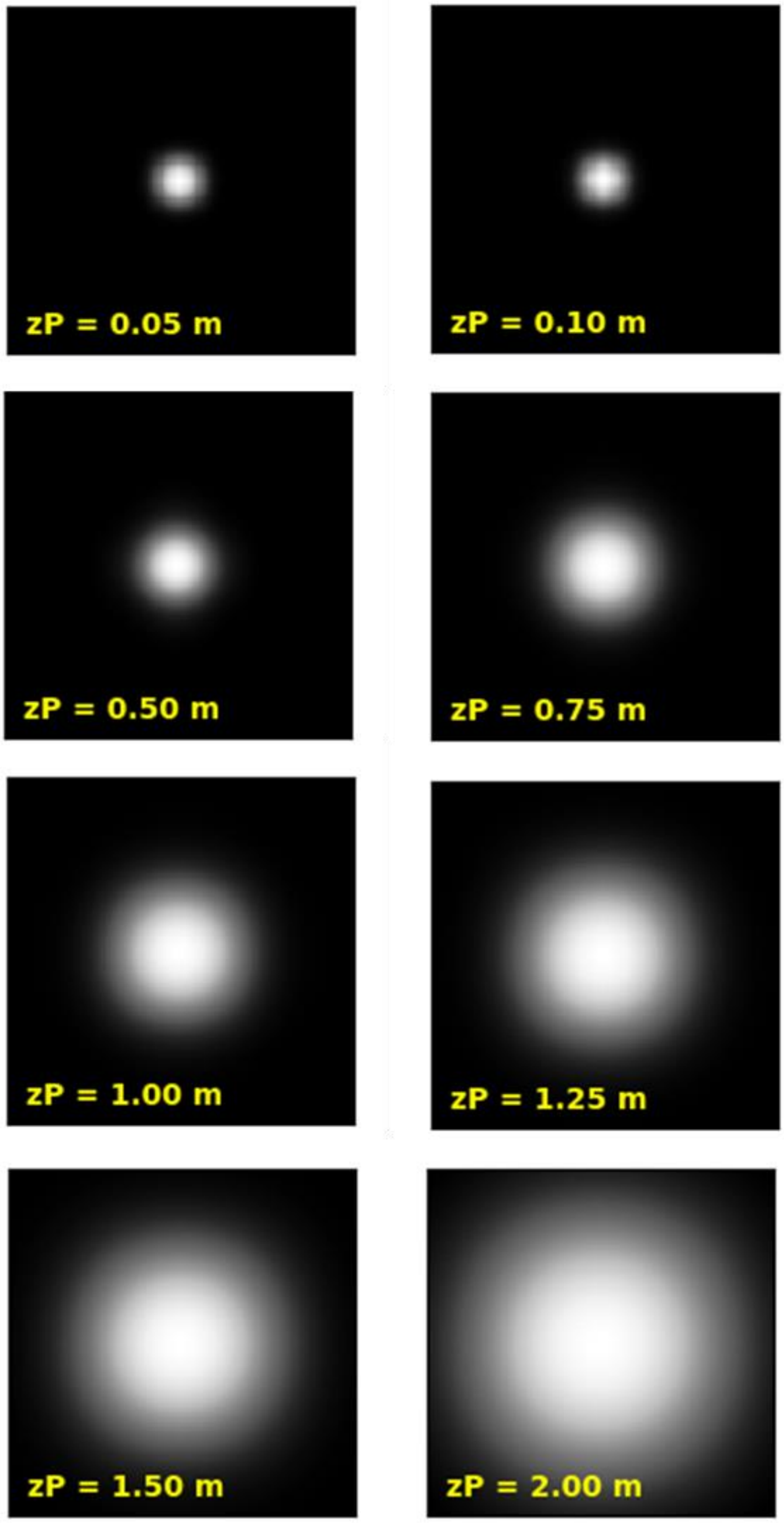


Fig. 5. The irradiance in XY planes for increasing values of zP.

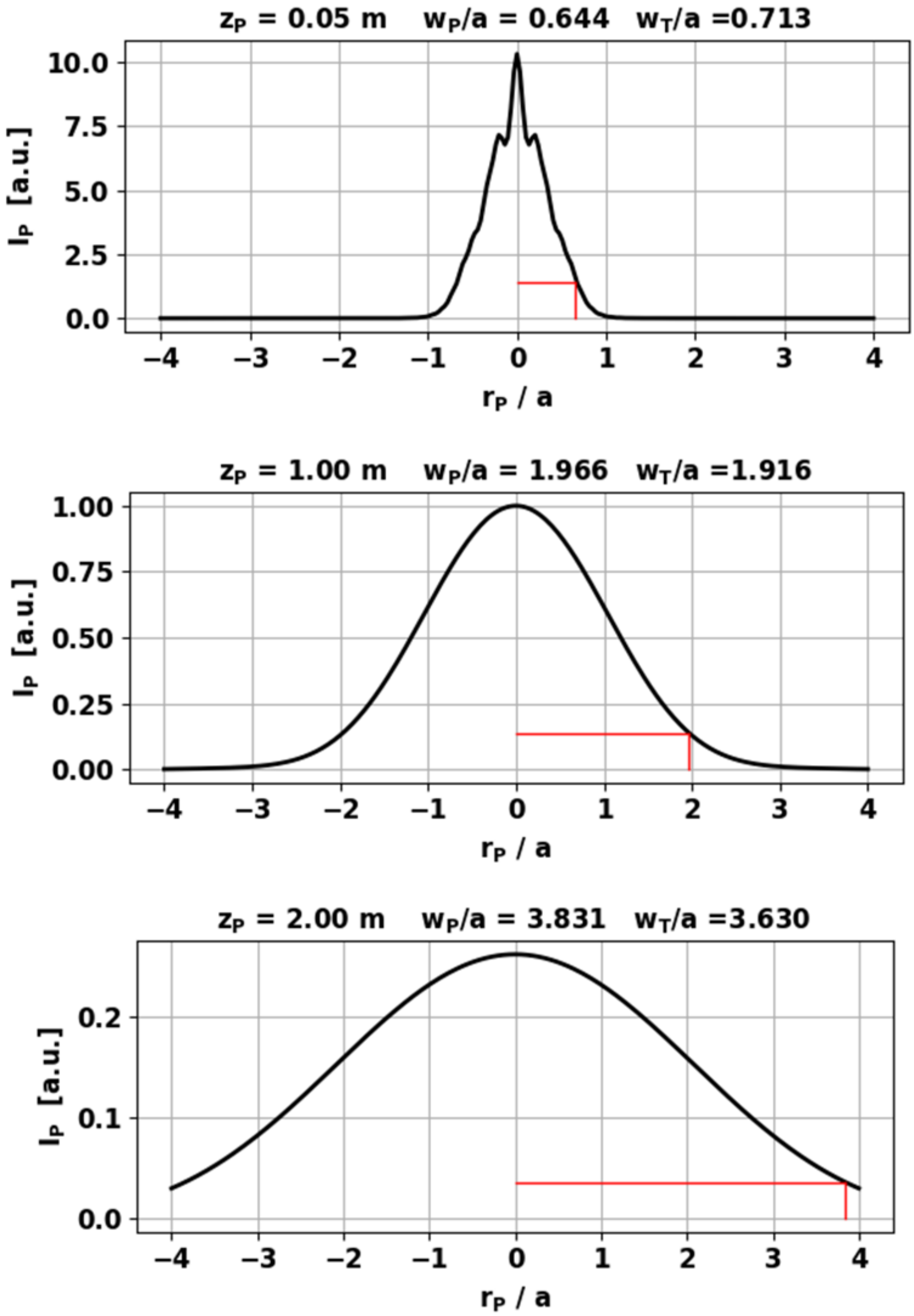
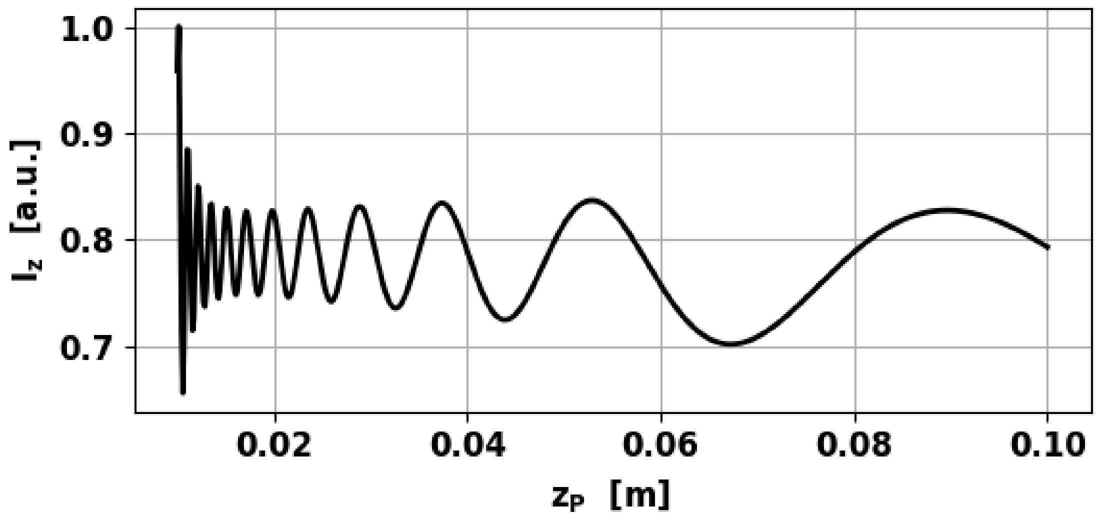


Fig. 6. Irradiance for three z_P values. As the distance from the aperture increases, the beam spot increases. w_T is the beam spot calculated from equation 1. In the far-field ($z_P > 1$ m), the peak intensity obeys the inverse square law to a good approximation.



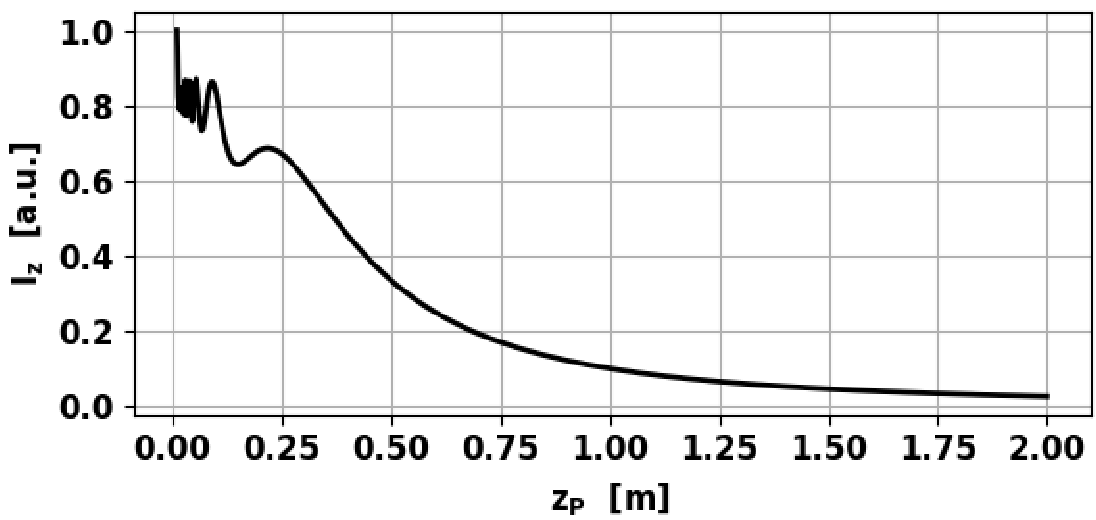


Fig. 7. The irradiance I_z along the optical axis (+Z axis). In the far-field the irradiance decreases according to the inverse square law.

$z_P = 1.00$ m $\quad I_z = 0.100$ a.u. $\qquad z_P = 2.00$ m $\quad I_z = 0.026$ a.u.

emRSGB01Z.py