DOING PHYSICS WITH PYTHON

COMPUTATIONAL OPTICS

INTERFERENCE

TWO COHERENT POINT SOURCES

Ian Cooper

matlabvisualphysics@gmail.com

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PYTHON CODES op003.py op003A.py op003B.py

INTERFERENCE

The theory of interference is based essentially on the principle of linear superposition of electromagnetic fields. When a number of waves arrive at the same location at the same time or exist together along the same direction, the fields produced at a point in empty is given by the vector sum.

$$\vec{E} = \sum_n \vec{E}_n \qquad \vec{B} = \sum_n \vec{B}_n$$

However, for the present we will consider unpolarized light or when the field vectors are parallel and treat the electric or magnetic field as scalar quantities.

This article will consider the interference produced by two point sources where harmonic wave disturbances propagate in free space in all directions from the sources. Spherical wavefronts (spherical surfaces of constant phase) emanate from each single point source. The wavefunction for the spherical wave emitted from a point source is given by

(1) $$u(\vec{r},t) = \frac{A}{r}\exp\left(i\left(k \bullet \vec{r} - \omega t + \phi\right)\right)$$

where r is the distance from the source. The irradiance I (intensity) of the spherical wave is

(2) $$I\left(\vec{r}\right) = u^*(\vec{r},t)u(\vec{r},t) = \left(\frac{A}{r}\right)^2$$

The two point sources are located along the Z axis: $S_1(0,0,+s)$ and $S_2(0,0,-s)$. The detector point is $P(x,y,z)$ and the distance from S_1 to P is R_1 and the distance from S_2 to P is R_2 (figure 1).

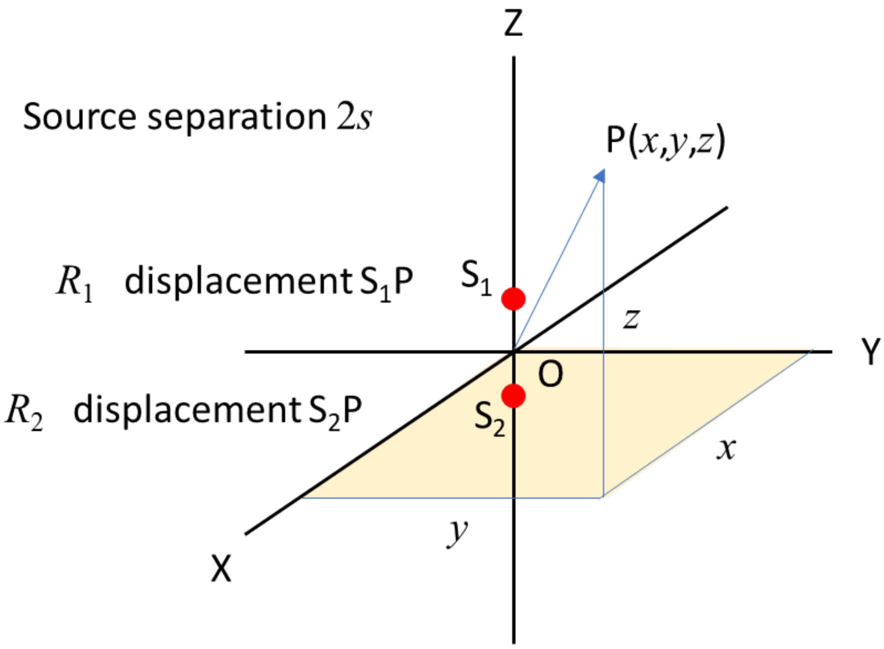

Fig. 1. Geometry for the interference computations from two source points S_1 and S_2. P is an observation point.

Equation 1 can be expressed as

(3) $$u(\vec{r},t) = \frac{A}{r}\exp\left(i\left(k\bullet\vec{r}+\phi\right)\right)\exp\left(i\left(-\omega t\right)\right)$$

So, the spatial variation in the wavefunction is

(4) $$u(\vec{r}) = \frac{A}{r}\exp\left(i\left(k\bullet\vec{r}+\phi\right)\right)$$

and at each point the electromagnetic fields will oscillate sinusoidally with frequency ω. However, the irradiance is time independent. Therefore, in modeling the interference produced by the two point sources, we only need to consider the spatial dependence of the wavefunction $u\left(\vec{r}\right)$ given by equation (4).

The Python code **op003.py** is used to calculate the wavefunction and irradiance in the XY plane (z = constant) for the arrangement shown in figure 1.

SINGLE POINT SOURCE

$\lambda = 500$ nm, $x_S = 0$, $y_S = 0$, $z_S = 0$, $s = 0$

wL [nm]	A	f [Hz]	T [fs]	phi/pi
500	1.0e-05	6.000e+14	1.667	0.00
500	1.0e-05	6.000e+14	1.667	0.00

The wavefunction and the irradiance is computed along the X axis Figure 2 shows the electric field with decreasing amplitude with distance from the source at time t = 0. The wavefunction varies sinusoidally with time at each detector point P.

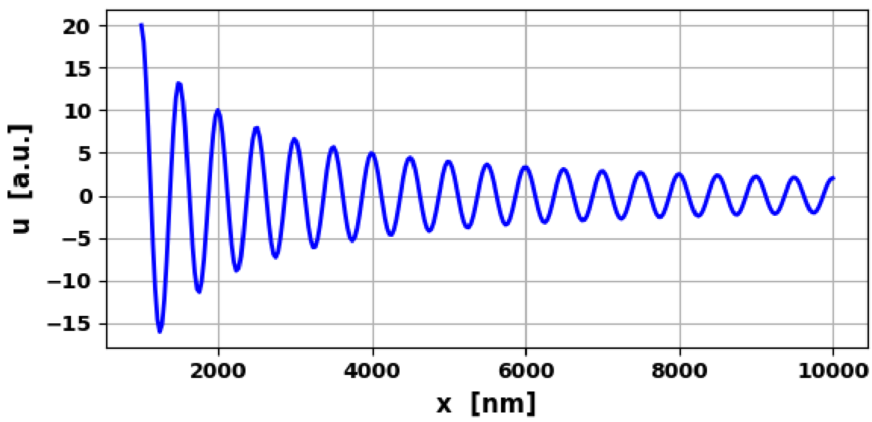

Fig. 2. Wavefunction in a radial direction in the plane z = 0. The distance between the peaks is equal to one wavelength, $\lambda = 500$ nm.

op003.py

Figure 3 shows the variation in the irradiance with distance from the source. The irradiance follows the inverse square law $I \propto 1/R^2$. The irradiance is independent of time.

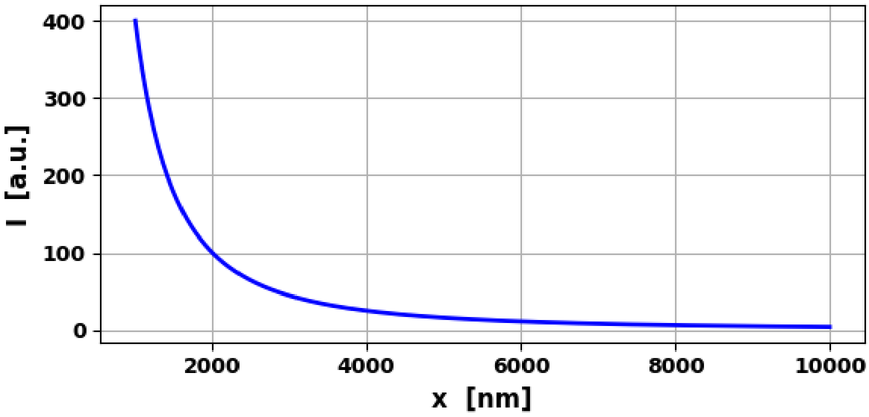

Fig. 3. Irradiance in a radial direction in the plane z = 0.

Inverse square law:

$$x_P = 2000\ \text{nm} \quad I_P = 100 \quad x_P = 4000\ \text{nm} \quad I_P = 25\,.$$

op003.py

Single point source radiating at two different wavelengths

$\lambda_1 = 500\ \text{nm},\ \ \lambda_2 = 250\ \text{nm},\ \ x_S\ =\ 0,\ y_S\ =\ 0,\ z_S\ =\ 0,\ s = 0$

wL [nm]	A	f [Hz]	T [fs]	phi/pi
500	1.0e-05	6.000e+14	1.667	0.00
250	1.0e-05	1.200e+15	0.833	0.00

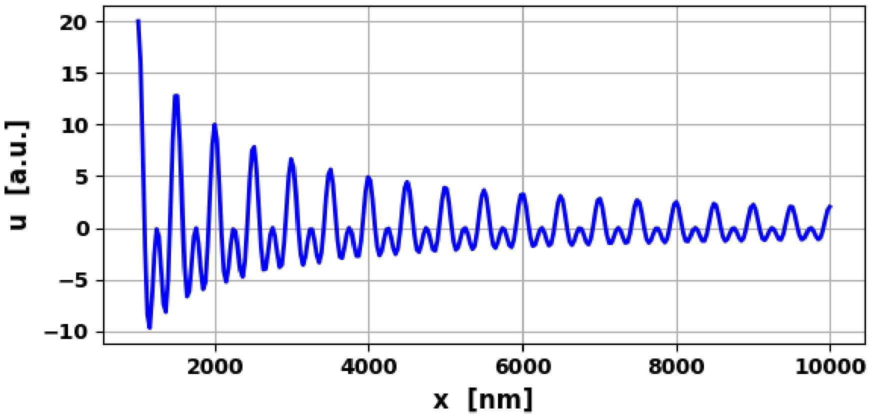

Fig. 4. Wavefunction in a radial direction in the plane z = 0. Peaks occur at intervals of 250 nm and 500 nm.

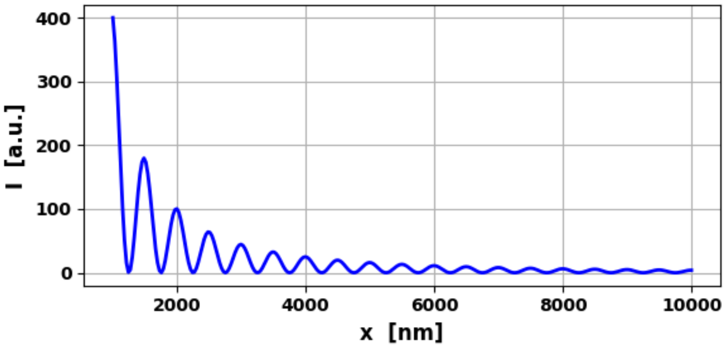

Fig. 5. Irradiance in a radial direction in the plane z = 0.

TWO COHERENT POINT SOURCE INTERFERENCE

The Python code **op003.py** is used to calculate the wavefunction and irradiance in a radial direction (X axis) in the XY plane, $z = 0$ from the two coherent point sources (figure 1). The pathlengths from the two source points to a detector point on the X axis are the same, $R_1 = R_2$. Hence, there is no interference between the two waves and the irradiance decreases with distance from the origin O for all detector points in the XY plane, $z = 0$. When the distance from the origin O, is large, the two source approximate a single source and the irradiance falls according to the inverse square law.

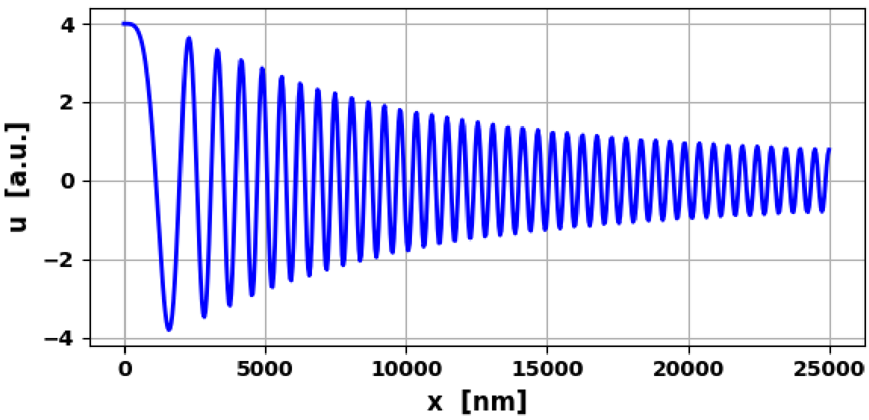

Fig. 6. Wavefunction in a radial direction in the plane $z = 0$.

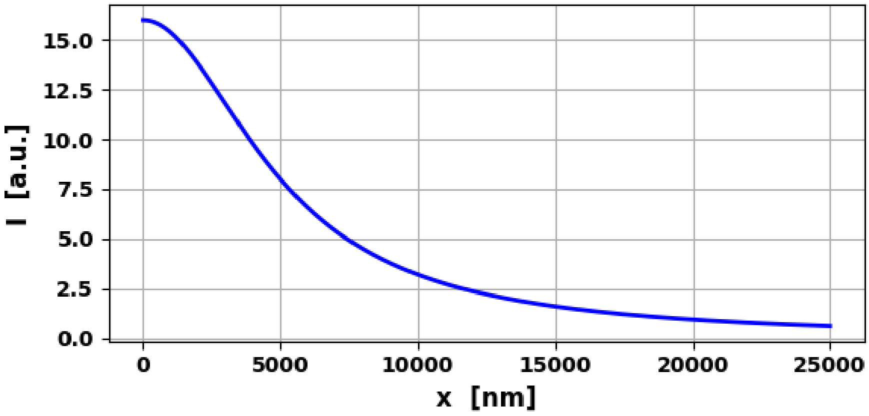

Fig. 7. Irradiance in a radial direction in the plane z = 0.

We can compute the interference pattern in a XY plane from the two point sources as shown in figure 8 using the Python code **op003B.py**.

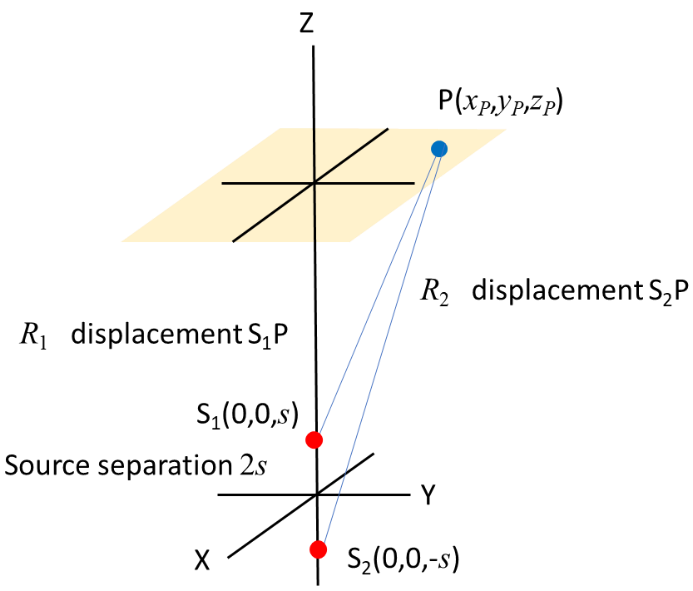

Fig. 8. Geometry for computing the wavefunction and irradiance in an XY plane.

wL [nm]	A	f [Hz]	T [fs]	phi/pi
500	1.0e-05	6.000e+14	1.667	0.00
500	1.0e-05	6.000e+14	1.667	0.00

source separation 2s/wL = 20.0

XY plane zP/wL = 100

The interference pattern in the XY plane with $z = 100\ \lambda$ is a set of concentric circular fringes centered on the Z axis (figure 9). Bright circles occur when the pathlength difference between the two waves is $m\lambda \quad (m = 0, 1, 2, ...)$. The dark circular regions occur when a crest from one source intersects a trough from the other source (waves out of phase: path difference $\left(m + \frac{1}{2}\right)\lambda \quad m = 0, 1, 2, ...$) due to destructive interference.

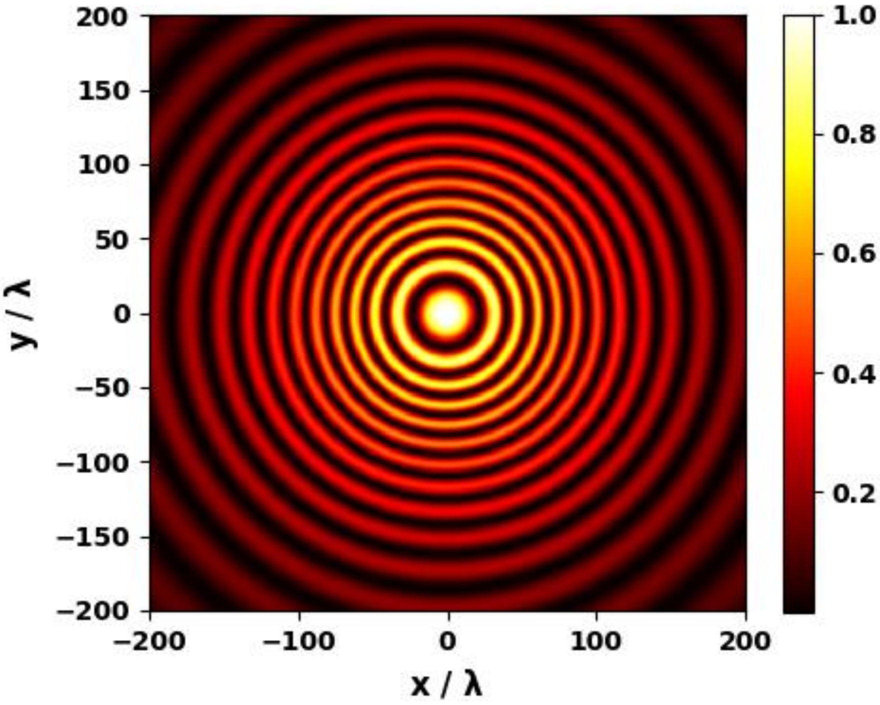

Fig. 9. Interference pattern in the XY plane, $z = 100\ \lambda$. Concentric circular fringes centered on the Z axis are observed. **op003B.py**

We can compute the interference pattern in a YZ plane from the two point sources as shown in figure 10 using the python code **op003A.py**.

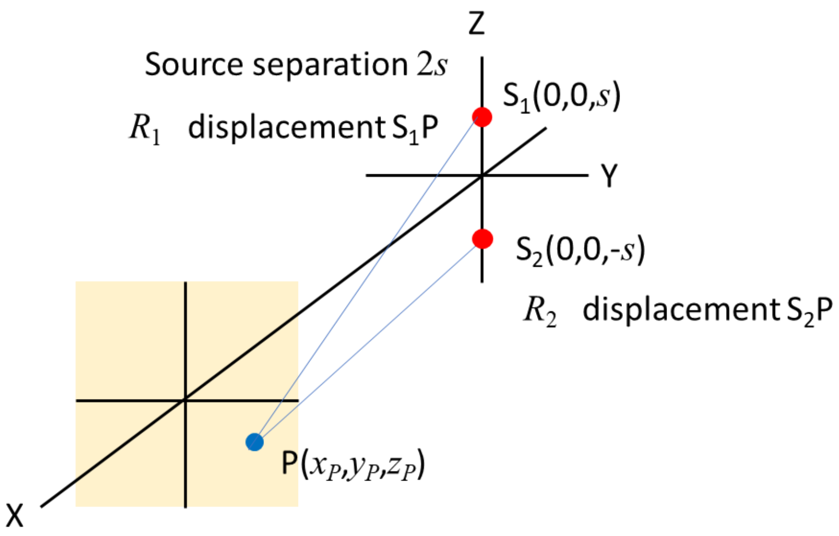

Fig. 10. Geometry for the two point interference pattern in an YZ plane.

wL [nm]	A	f [Hz]	T [fs]	phi/pi
500	1.0e-05	6.000e+14	1.667	0.00
500	1.0e-05	6.000e+14	1.667	0.00

source separation 2s/wL = 20.0

YZ plane xP/wL1 = 100 m

The crests and troughs of the spherical waves spread-out from the source points. Bright regions occur when the crests or troughs meet (waves in phase: path difference $m\lambda \quad m = 0, 1, 2, \ldots$) due to constructive interference. Dark regions occur when a crest from one source intersects a trough from the other source (waves out of phase: path difference $\left(m + \frac{1}{2}\right)\lambda \quad m = 0, 1, 2, \ldots$) due to destructive interference. Surrounding the two point sources a fringe pattern of bright and dark regions exists. Figure 11 shows the fringe pattern in the YZ plane $x = 100\lambda$. For a [3D] perspective, think around rotating the fringe pattern around the Z axis where the lines of maximum and minimum give a hyperbolic surface.

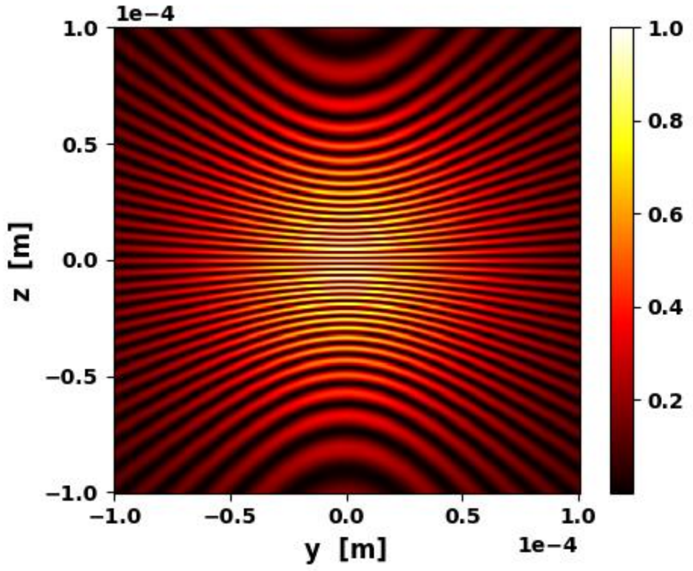

Fig. 11. Bright and dark fringes for two coherent point sources. Hyperbolic surfaces are generated by rotating the pattern about the Z axis. **op003A.py**