RADIOACTIVE DECAY SIMULATION

When does a radioactive nucleus decay?

The simulation activity

A sample of a radioactive isotope consists of an enormous number of unstable nuclei. These unstable nuclei change by emitting an alpha particle, beta particle or gamma ray. The unstable nucleus is called a **parent**. It decays to form a **daughter** nucleus. The death of the parent gives birth to the daughter. The decay of any parent nucleus is a **random** process. It is not possible to predict which nucleus will decay, its survival is determined by the law of chance. However, it is possible to state the **probability** that a nucleus will decay in a given time interval. This probability that an unstable nucleus will decay spontaneously is:

* independent of the parent nucleus' history,
* the same for each nucleus of the same type,
* very nearly independent of external influences (pressure, temperature, chemical bonding, etc.).

We can simulate the radioactive decay process by using a set of 100 numbers from 0 to 99. A list of the 100 numbers or a set of 100 numbered cards can be used. Each number represents a nucleus. A number selected at random from 0 to 99 indicates that this numbered nucleus has decayed and its number is crossed off the list or removed from the cards. The random numbers can be chosen from reading the last two digits of a telephone number from any page of a telephone directory. This gives the probability of a nucleus decaying to be 1/100. The selection of each number represents one time interval. Record the number chosen and the number of nuclei remaining after each random number has been selected. To simulate a radioisotope with a shorter half-life, choose three numbers at random and delete these numbers from the list or remove there cards.

The results can then be analysed several ways:

* Results plotted on linear graph paper: no. of nuclei vs. time. The half-life can be determined by finding the time it takes for the number of nuclei to half.
* Results plotted on log/linear graph paper: ln(no. of nuclei) vs time. The slope of the graph gives the decay constant.
* Results analysed using various computer data analysis programs, for example spreadsheets.
* The average lifetime can be determined by recording the time it takes for each nucleus to decay, and then finding the average of this set of numbers.**Theory of Radioactive Decay**

The decay of a nucleus cannot be predicted. But, we can state the probability that a nucleus will decay. Let p be the probability of a nucleus decaying in a small time interval dt. The probability that it will decay is directly proportional to this time interval

$$p = \lambda \, dt$$

where the proportionality constant λ is called the **decay constant**.

Since the probability of a nucleus decaying or surviving is 1,

probability of a nucleus decaying = p
probability of a nucleus surviving = (1 - p)

If N_o is the initial number of nuclei, then after one time interval,

number of nuclei decaying $= pN_o$
number of nuclei surviving $= (1 - p)N_o$

and after two time intervals,

number of nuclei decaying $= p(1 - p)N_o$
number of nuclei surviving $= (1 - p)N_o - p(1 - p)N_o = (1 - p)^2 N_o$

Therefore, after t time intervals, the number of nuclei surviving is

$$N = (1 - p)^t . N_o \tag{1}$$

The decay of a radioisotope obeys the exponential law

$$N = N_o e^{-\lambda t} \tag{2}$$

From equations (1) and (2), the relationship between the probability and decay constant is

$$\lambda = -\ln(1 - p) \tag{3}$$
$$p = 1 - e^{-\lambda} \tag{4}$$

The rapidity of decay is measured by the **half-life**, which is the time it takes for half the original number of unstable nuclei to decay.

Using equation (2)

$$\tfrac{1}{2}N_o = N_o e^{-\lambda T_{\frac{1}{2}}}$$

$$T_{\frac{1}{2}} = \frac{\ln(2)}{\quad} \tag{5}$$

$$T_{\frac{1}{2}} = -\ln(2)/\ln(1 - p) \tag{6}$$

The **average (mean) lifetime** of a nucleus is the reciprocal of the decay constant

$$T_{av} = \frac{1}{\lambda} = \frac{T^{1/2}}{\ln(2)} = -1/\ln(1 - p) \tag{7}$$

Another useful quantity describing radioactive decay is the **activity**. The activity A is number of decays in a given time interval. Using equation (2)

$$A = -dN/dt = \lambda N_o e^{-\lambda t} = \lambda N$$

$$A = A_o e^{-\lambda t} \tag{8}$$

where $A_o =\ \ N_o$. The minus indicates that the number of nuclei decreases with time.

Therefore, the activity:

* follows the same exponential law that describes the number of surviving nuclei,
* is proportional to the number of radioactive nuclei.

The SI unit for activity is the **becquerel** (Bq), which is one decay per second. A common unit for the activity is the **curie** (Ci). The curie is defined to be 3.7×10^{10} disintegrations per second.

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

Typical results

One random number chosen in each time interval:

Theoretical values

$p = 1/100 = 0.01$

equation (6) $T_{1/2} = 69$

equation (7) $\lambda = 0.010$

equation (7) $T_{av} = 99$

Experimental values

least squares fit $T_{1/2} = 67$

$\lambda = 0.010$

Three random numbers chosen in each time interval:

Theoretical values

$p = 3/100 = 0.03$

equation (6) $T_{1/2} = 23$

equation (7) $= 0.030$

equation (7) $T_{av} = 33$

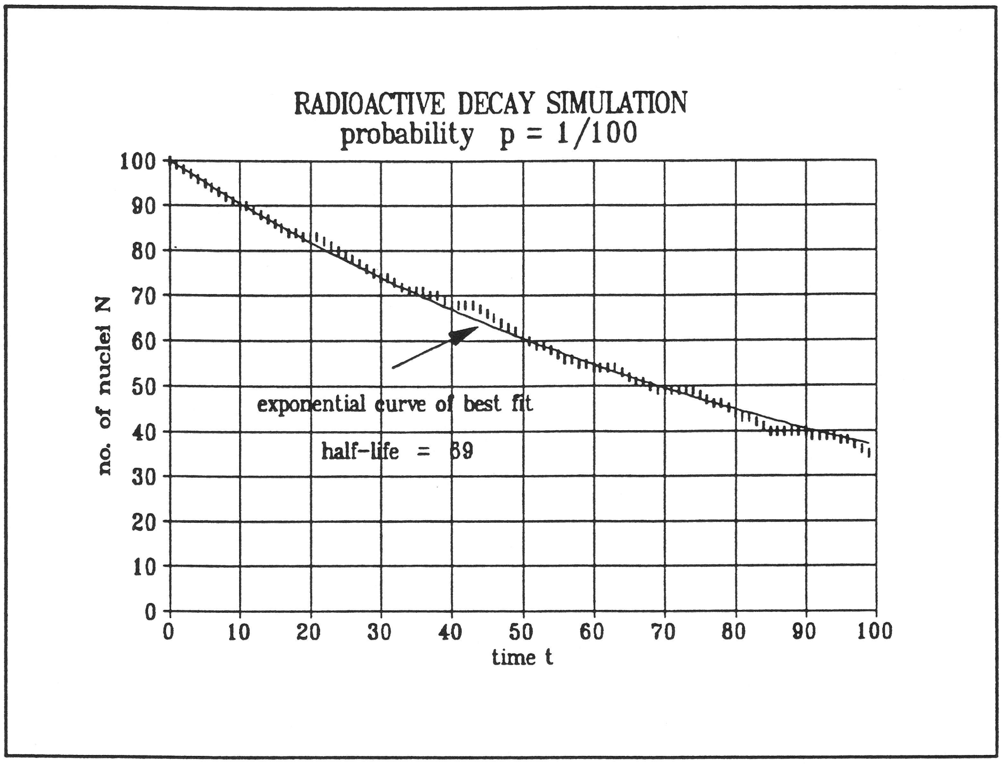
Experimental values

least squares fit $T_{1/2} = 24$

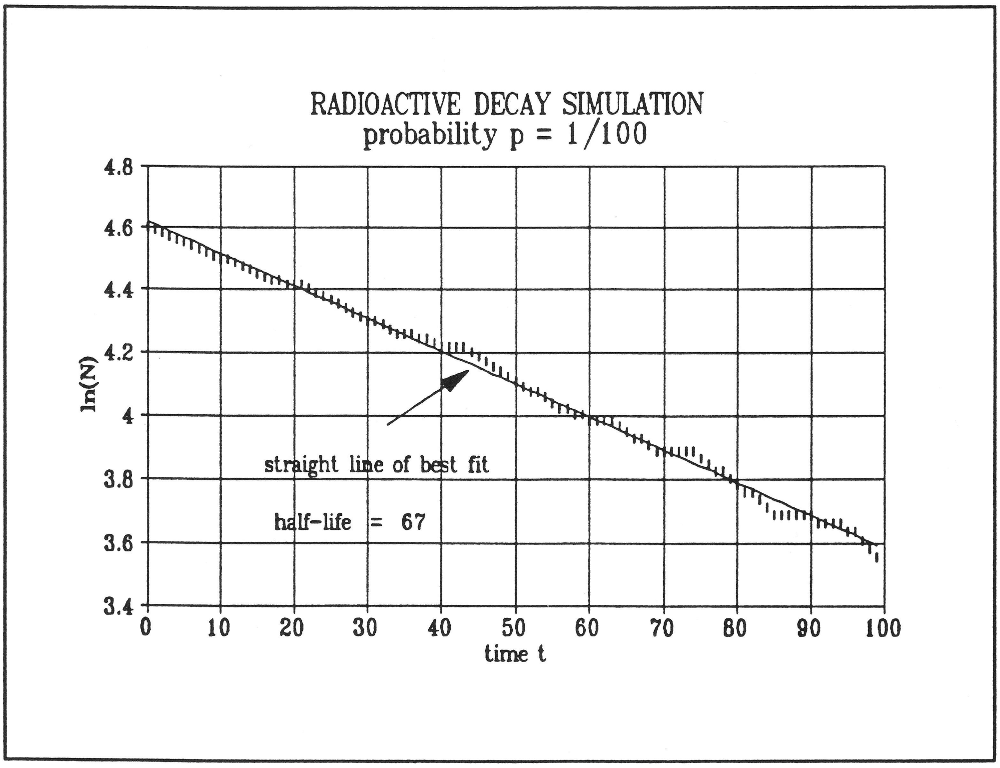
$\lambda = 0.029$

average lifetime $T_{av} = 35$

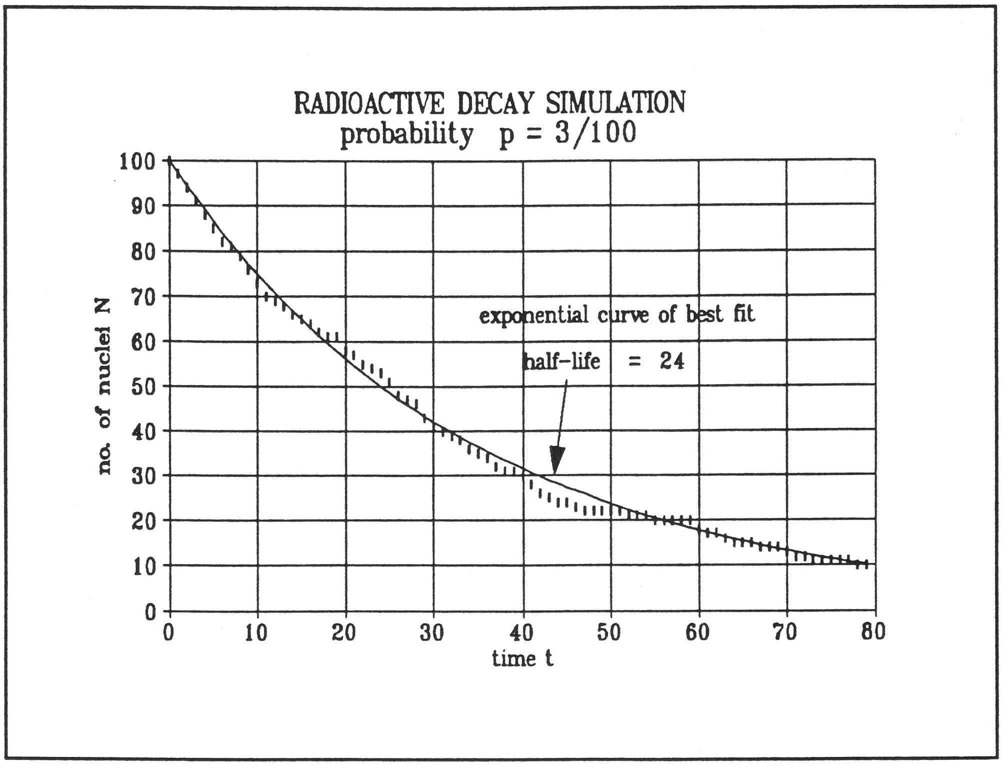
The results obtained from the simulation agree very well with the values predicted by the laws of chance.

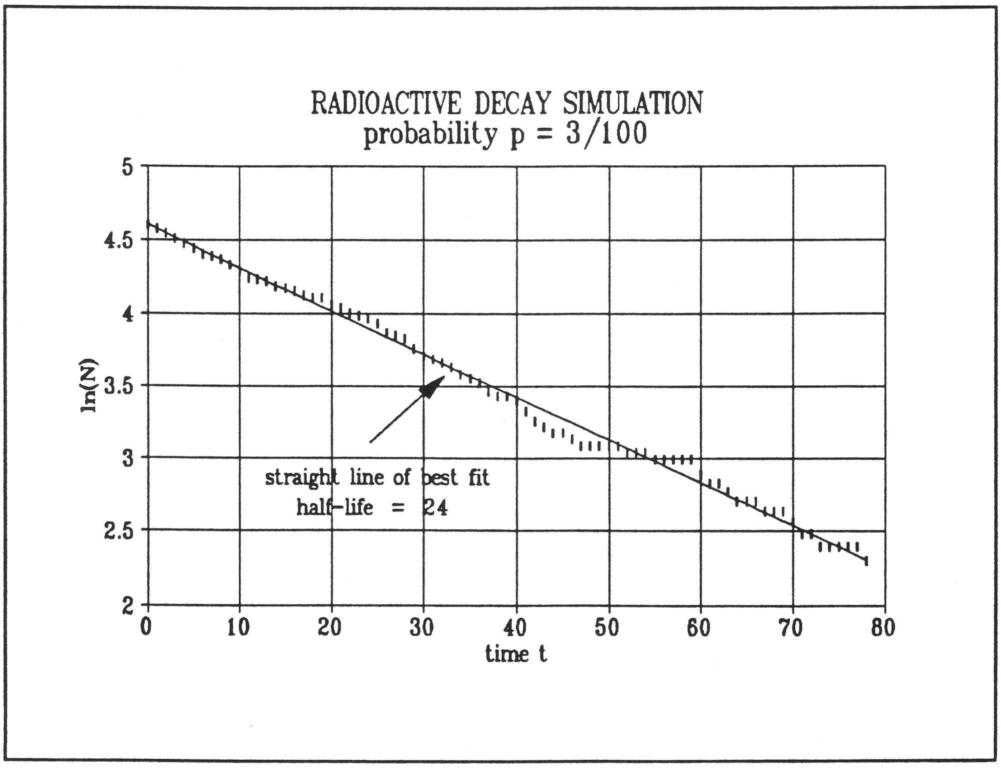


RADIOACTIVE DECAY SIMULATION
probability p = 1/100
no. of nuclei N
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
time t
exponential curve of best fit
half-life = 69



RADIOACTIVE DECAY SIMULATION
probability p = 1/100
ln(N)
4.8
4.6
4.4
4.2
4
3.8
3.6
3.4
0
10
20
30
40
50
60
70
80
90
100
time t
straight line of best fit
half-life = 67


RADIOACTIVE DECAY SIMULATION
probability p = 3/100
no. of nuclei N
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
time t
exponential curve of best fit
half-life = 24


RADIOACTIVE DECAY SIMULATION
probability p = 3/100
ln(N)
5
4.5
4
3.5
3
2.5
2
0
10
20
30
40
50
60
70
80
time t
straight line of best fit
half-life = 24

RADIOACTIVE DECAY ANALOG

0	10	20	30	40	50	60	70	80	90
1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99

0	10	20	30	40	50	60	70	80	90
1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99

RADIOACTIVE DECAY ANALOG

time	decays	nuclei N	time	decays	nuclei N	time	decays	nuclei N	time	decays	nuclei N

AVERAGE LIFETIME

nucleus	lifetime	nucleus	lifetime	nucleus	lifetime	nucleus	lifetime	nucleus	lifetime
00		20		40		60		80	
01		21		41		61		81	
02		22		42		62		82	
03		23		43		63		83	
04		24		44		64		84	
05		25		45		65		85	
06		26		46		66		86	
07		27		47		67		87	
08		28		48		68		88	
09		29		49		69		89	
10		30		50		70		90	
11		31		51		71		91	
12		32		52		72		92	
13		33		53		73		93	
14		34		54		74		94	
15		35		55		75		95	
16		36		56		76		96	
17		37		57		77		97	
18		38		58		78		98	
19		39		59		79		99	

average lifetime =